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A Peep into Multi-Objective Transportation Problem

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Abstract

A multi-objective programming is for transportation model with consideration of depot to customer relationships. In this paper we are going to analyze the multi-objective transportation problem. This paper is very useful to understand MOTP for upcoming researchers.

Keywords: *Transportation Problem, Linear programming, Optimization, fuzzy goal programming*

INTRODUCTION

The conservative transportation problem usually involves the transportation of commodities from several deliver points to different demand points and considers the minimization of the total transportation costs. Transportation problem (TP) is a unique case of linear programming problem (LPP) in which cost optimization has been made on the base of demand and resources. Optimization of cost is a common objective in TP. There is a great deal of work on various destinations by different analysts has been distributed with the end goal that limit shipping cost, least weakening while transportation, minimize delivery time, etc. Combing of more than two objectives are multi-objective transportation problem. Different approached for methods solving multi-objective transportation problem (MOTP) also discusses by various authors viz. goal programming problem, fuzzy techniques, genetic algorithm, etc.

Optimization deals with the study of those kinds of problems in which one has to minimize or maximize one or more objectives that are functions of some real or integer variables. This is executed in a

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systematic way by choosing the proper values of real or integer variables within an allowed set. Given a defined domain, the main goal of optimization is to study the means of obtaining the best value of some objective function.

Single-Objective Optimization Problem

In numerous genuine circumstances issues are demonstrated and tackled as single-objective optimization problems in a deterministic and fresh condition. The general form of single-objective optimization problems is:

Minimize or Maximize $f(x,)\ \ X=(x_1,x_2,...,x_n)$

Subject to

$$g_j(X,)\le 0,\quad j=1,2,...,k$$

$$l_j(X,)\ge 0,\quad j=1,2,...,r$$

$$h_j(X,)=0,\quad j=1,2,...,m$$

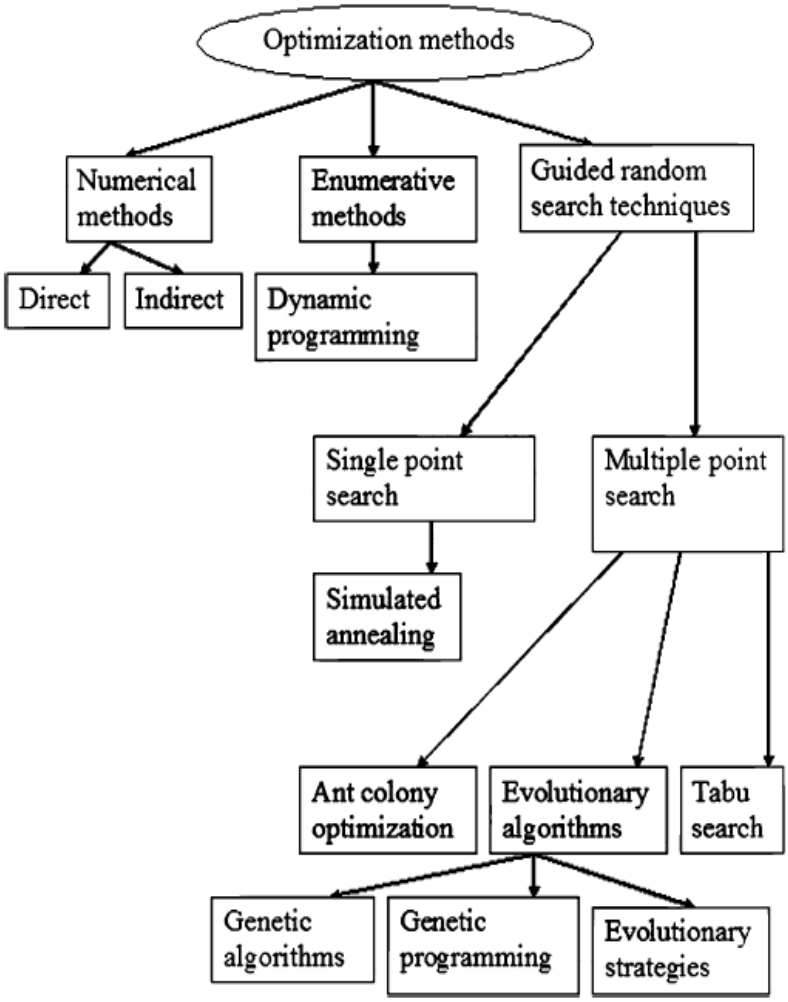


Fig.1 The different search and optimization techniques

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Where $f, g_1, g_2, ...g_k, l_1, l_2, ...l_r, h_1, h_2, ...h_m$ are real valued functions defined on R_n $X = (x_1, x_2, ..., x_n) \in R^n$ is called decision vector and $x_1, x_2, ..., x_n$ are called decision or unknown variables. In case all the functions (objective function constraints) are linear then the above problems is called linear programming problem, otherwise it is called non-linear programming problem.

Multi-objective Optimization Problem:

The mathematical definition of a multi objective problem (MOP) is important in providing a foundation of understanding between the interdisciplinary nature of deriving possible solution techniques (deterministic, stochastic); i.e., search algorithms. The single objective formulation is extended to reflect the nature of multi objective problems where there is not one objective function to optimize, but many. Thus, there is not one unique solution but a set of solutions. This set of solutions is found through the use of Pareto Optimality Theory [19].

The most general mathematical model of amulti-objective optimization problem is:

$$Max. \quad f(x,) = (f_1(x), f_2(x), ...f_m(x)) \quad ;$$

$$X = (x_1, x_2, ...x_n)$$

Subject to:

$$g_j(X,) \le 0, \quad j = 1,2,..., k$$

$$l_j(X,) \ge 0, \quad j = 1,2,..., r$$

$$h_j(X,) = 0, \quad j = 1,2,..., m$$

Where $f_1, f_2, ... f_m$ are the objective functions, Variables $x_1, x_2, ...x_n$ are called decision variables and X is called decision vector. This problem is also called multi-objective programming problem.

Transportation Problems:

The Transportation Problem (TP) was first developed and proposed by F. L. Hitchcock since 1941. It usually aims to minimize the total transportation cost. Other objectives that can be set are a minimization of the total delivery time, a maximization of the profit, etc. The Hitchcock-Koopman's transportation problem is expressed as a linear transportation model as follows:

Minimize $f(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_{i,} \, i = 1,2,....., m \quad (Supply)$$

$$\sum_{i=1}^{m} x_{ij} = b_{j,} \, j = 1,2,....., n \quad (Demand)$$

$$x_{ij} \ge 0 \quad for\ all\ i\ and\ j$$

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Multi-objective Transportation Problem (MOTP):

The transportation problem (TP) is a special type of linear programming. It will be considered as a minimum cost flow problem. That is, we want to transfer a quantity of products from plants to warehouses to minimize the transportation cost. TP usually involves an objective function or multiple objective functions. This last type of problem is called multi-objective transportation problem (MOTP). The multi-objective transportation problem with k objectives can be represented as:

$$\min f_1(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c^1{}_{ij}\, x_{ij}$$

$$\min f_2(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c^2{}_{ij}\, x_{ij}$$

:

$$\min f_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c^k{}_{ij}\, x_{ij}$$

Subject to $\sum_{j=1}^{n} x_{ij} \le a_{i,}\, i = 1,2,.....,m \quad$ *for all i.*

$$\sum_{i=1}^{m} x_{ij} = b_{j,}\, j = 1,2,.....,n \quad \textit{for all } j.$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ and } x_{ij} \ge 0 \quad \textit{for all } i \textit{ and } j$$

Where c_{ij}^{k} represents the coefficients related to x_{ij} variable for objective k.

MOTP with Equality and Inequality Constraints

MOTP with equality constraint is of the form:

$$\min f_r(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c^r{}_{ij}\, x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_{i,}\, i = 1,2,.....,m \quad \textit{for all } i.$$

$$\sum_{i=1}^{m} x_{ij} = b_{j,}\, j = 1,2,.....,n \quad \textit{for all } j.$$

The above problem is feasible if and only if the condition

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ and } x_{ij} \ge 0 \quad \textit{for all } i \textit{ and } j$$

MOTP with inequalities both in supply and demand constraints can be presented as:

$$\min f_r(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c^r{}_{ij}\, x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \le a_{i,}\, i = 1,2,.....,m \quad \textit{for all } i.$$

$$\sum_{i=1}^{m} x_{ij} \ge b_{j,}\, j = 1,2,.....,n \quad \textit{for all } j.$$

This problem is feasible if and only if

$$\sum_{i=1}^{m} a_i \ge \sum_{j=1}^{n} b_j \text{ and } x_{ij} \ge 0 \quad \textit{for all } i \textit{ and } j$$

Some works have been done in transportation problem:

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Wahed and Lee (21) proposed a natural fuzzy goal programming approach to manage choice the favored deal respond in due order regarding target transportation issue. The proposed approach considers the free thought of the data by executing the base overseer and moreover expects that each objective work has a fuzzy goal. The approach centers on limiting the most exceedingly terrible upper bound to acquire a proficient arrangement which is near the best lower bound of every goal work. The solution procedure controls the search direction via updating both the membership values and the aspiration levels..Zangiabadi and Maleki (23) exhibited a fuzzy goal programming way to deal with decide an ideal trade off answer for the multi-objective transportation issue by accepting that every target work has a fluffy objective. A unique sort of non-linear (hyperbolic) enrollment work is appointed to every target capacity to depict each fuzzy goal..

Surapati and Roy (20) presented a precedence based fuzzy goal programming approach for solving a multi-objective transportation problem with fuzzy coefficients. Initially, they defined the relationship functions for the fuzzy goals. Consequently, they transformed the relationship functions into relationship goals, by assigning the highest degree (unity) of a relationship function as the ambition level andintroducing deviational variables to each of them. In the solution process, negative deviational variables are minimized to obtain the most satisfying solution. Osuji et al (14) carried out an investigation on the inconsistency algorithm function of linear transportation problem. Two mathematical examples were used for the study. In their paper, an well-organized algorithm for solving a linear programming problem was clearly discussed, and it was concluded that paradox does not exist in the first set of data, while paradox existed in the second set of data. The Vogel's Approximation Method (VAM) was used to obtain the initial basic feasible solution via the Statistical Software Package known as TORA. The first set of data exposed that paradox does not exist, while the second set of data showed that paradox exists. The method however gave a step bystep development of the solution procedure for finding allthe paradoxical pair in the second set of data.

CONCLUSION

Problems with multiple objectives arise in a natural fashion in most disciplines and their solution has been a challenge to researchers for a long time. Despite the considerable variety of techniques

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developed in Operations Research(OR) and other disciplines to tackle these problems, the complexities of their solution calls for alternative approaches. In this work we have developed the introductory part of Multi-objective Transportation Problem so that the new upcoming researcher can easily understand the topic.

REFERENCES

I. Andrea Raith, Multiobjective routing and transportation problem thesis, 2009.

II. C. T. Chen, "A fuzzy approach to select the location of the distribution center," Fuzzy Sets and Systems, vol. 118, pp. 65-73, 2001.

III. Dr Kurt Fedra,"Sustainable urban transportation",the project SUTRA(EVK4-CT-1999-00013), pp.1-36.

IV. E.E. Ammar and E.A.Youness," Study on multiobjective transportation problem with fuzzy numbers with fuzzy numbers", Applied Marhematics and computation ,Vol. 166, pp. 193-204,2001.

V. J. K. Sharma, Operations Research Theory and Applications, Macmillian India Ltd book second edition 2003.

VI. J. Korpela, A. Lehmusvaara, and M. Tuominen, "Customer service based design of the supply chain," International Journal of Production Economics, vol. 69, pp. 193-204, 2001.

VII. J. P. Ignizio, Linear programming in single- & multiple-objective system: Prentice-Hall, Inc., 1982.

VIII. K. C. Rao and S. L. Mishra, Operations research: Alpha Science International Ltd., 2005.

IX. L. Li and K. K. Lai, "A fuzzy approach to the multiobjective transportation problem," Computers & Operations Research vol. 27,pp. 43-57, 2000.

X. M. Ehrgott. Multicriteria Optimization. Springer, Berlin, second edition2005. ISBN 3-540-21398-8

XI. M. Sakawa, I. Nishisaki, and Y.Uemura, "Fuzzy programming

Page 85-92 © MANTECH PUBLICATIONS 2018. All Rights Reserved

and profit and cost allocation for a production and transportation problem," European Journal of Operational Research, vol. 131, pp. 1-15, 2001.

XII. M. Zeleny, Multiple criteria decision making: McGraw-Hill Book Company, 1982.

XIII. M.S. Chen , J. Han, P.S.Yu, Data Mining: "An overview from a Database Perspective" , IEEE Transactions on Knowledge and Data Engineering, Vol. 8, No.6, pp. 866-883,1996.

XIV. Osuji G. A. Opara J., Nwobi A. C., Onyeze V., Iheagwara A. I.(2013): Paradox Algorithm in Application of a Linear Transportation Problem. American Journal of Applied Mathematics and Statistics, 2014, Vol. 2, No. 1,10-15. DOI:10.12691/ajams-2-1-3

XV. P. Herabat and A. Tangphaisankun,"Multi-Objective Optimization Model using Constraint-Based Genetic Algorithms for Thailand Pavement Management" Journal of the Eastern Asia Society for Transportation Studies, Vol. 6, pp. 1137 - 1152, 2005.

XVI. S. Chanas and D. Kuchta, "A concept of the optimal solution of the transportation problem with fuzzy cost coefficients," Fuzzy Sets and Systems, vol. 82, p. 299-305, 1996.

XVII. S. Chanas and D. Kuchta, "Fuzzy integer transportation problem,"Fuzzy Sets and Systems, vol. 98, pp. 291-298, 1998.

XVIII. S. Lawrence, K. Bollacker, C. L Giles, " Indexing and Retrieval of Scientific Literature"Eight International Conference on Information and Knowledge Management, pp. 139-146, 1999.

XIX. S. T. Liu and C. Kao, "Solving fuzzy transportation problems based on extension principle," European Journal of Operational Research,vol. 153, pp. 661-674, 2004.

XX. Surapati, P. and Roy, T. K. (2008). Multi-objective transportation model with fuzzy parameters: Priority based fuzzy goal programming approach, Journal of

Page 85-92 © MANTECH PUBLICATIONS 2018. All Rights Reserved

Transportation System Engineering and Informational Technology, Vol. 8, 2008, pp 40-48

XXI. Wahed, W. F. and Lee, S. M. (2006). Interactive fuzzy goal programming for multi-objective transportation problems, Omega, Vol. 34, 2006, pp. 158-166.

XXII. WuttinanNunkaew and BusabaPhruksaphanrat, "AMultiobjective Programming for Transportation Problem with the Consideration of both Depot to Customer and Customer to Customer Relationships", the International Multi Conference of Engineers and Computer Scientists Vol IIIMECS 2009.

XXIII. Zangiabadi, M. and Maleki, H. R. (2007). Fuzzy goal programming for multi-objective transportation problems, Applied Mathematics Computation, Vol. 24-2007, pp. 449- 460.

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