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The Field-Dependent Lorentz Transformation as the Foundation for the Equivalence Principle

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Abstract

Within the scope of the equivalence principle, the equality of gravitational and inertial mass was examined. The gravitational mass is described in terms of velocity and potential using velocity invariance and the Lorentz transformation of curved space. Utilizing the same transformations, inertial mass was expressed in terms of acceleration and potential. Gravitational mass and inertial mass are demonstrated to be equivalent.

Keywords: *Gravitational mass, Equivalence Principle, Transformation, Curved Space*

INTRODUCTION

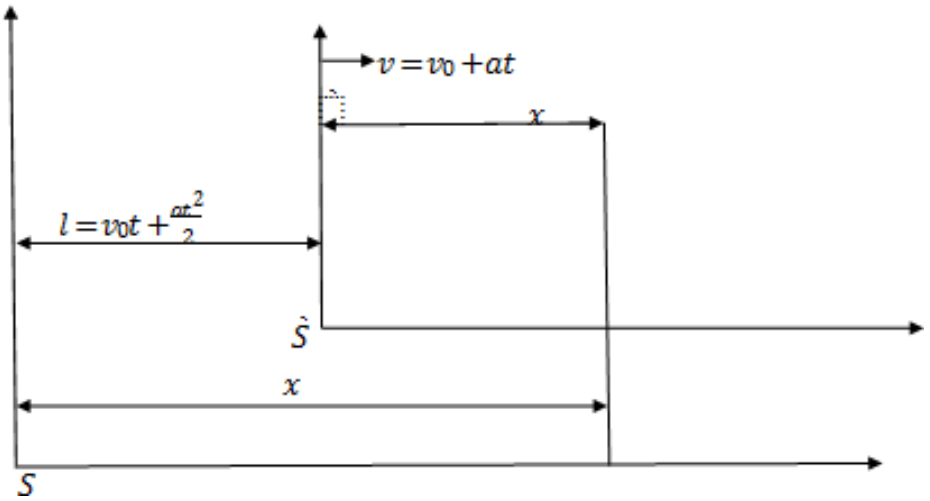
Equivalence principle and inertia are connected. Galileo formulated the equivalence principle when he discovered that the acceleration of a mass owing to gravitation is independent of the mass being accelerated. Galileo achieved his discoveries by rolling balls down almost frictionless inclined planes to reduce the velocity and improve timing precision. Loránd Eotvos has conducted increasingly accurate tests with the torsion balance pendulum. From Newton's second law, one may estimate the inertial mass owing to the force that accelerates the body, and from Newton's equation of gravity, one can estimate the gravitational mass. Albert Einstein founded his general theory of relativity on the premise that the connection between inertial and gravitational mass is not coincidental. However, according to his idea, gravity is not a force and is thus not subject to Newton's third rule; hence, the equivalence

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of inertial and gravitational mass remains baffling. Einstein further mentioned two reference frames, S and S. So that objects in both frames experience similar forces, S is a uniform gravitational field, whereas S is uniformly accelerated. Before developing the idea of curved space-time, Einstein linked the equivalence principle with special relativity to predict that time is altered by a gravitational potential and light beams bend in a gravitational field. Einstein's original description of the equivalence principle indicated that freefall and inertial motion were physically comparable. The inertial mass of an item is determined by its resistance to acceleration. In special relativity, there are two types of mass: rest mass and relativistic mass, which rises with velocity. This notion has been championed by Ernst Mach and refined by Percy W. Bridgman [3,6]. Rest mass is the Newtonian mass as determined by a stationary observer. The mass of a body or system is related to its entire amount of energy. However, there is no name that describes the influence of gravity or other fields on mass. This study investigates the influence of the gravitational field and acceleration on mass. Then, the equivalence principle may be verified.

THE EQUIVALENCE PRINCIPLE ON THE BASIS OF AVERAGE VELOCITY LORENTZ TRANSFORMATION

The equivalence principle argues that the laws of nature have the same shape in both accelerated and gravitationally permeating frames. This may be explored in the context of the Lorentz transformation based on the assumption of average velocity, as proposed by M.Dirar and others.



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$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{\frac{m}{c^2}}}} \quad (1)$$

Where the average velocity which was assumed to be invariant

$$v_m = \frac{v_0 + v}{2} (2)$$

For particle moving with initial velocity v_0 against gravity, the final velocity is given according to the ordinary rectilinear motion with constant acceleration to be

$$v^2 = v_0^2 - 2ax = v_0^2 - 2\varphi \quad (3)$$

Where φ stands for the potential per unit mass. Thus according to equation (3), (2) and (1)

$$v_m^2 = \frac{v^2 + 2vv + v^2}{4}$$

$$= \frac{2v^2 + 2\varphi + 2v\sqrt{v^2 + 2\varphi}}{4}$$

$$= \frac{v^2 + \varphi + v\sqrt{v^2 + 2\varphi}}{2} \quad (4)$$

Thus equation (1) become

$$m = \frac{m_0}{\sqrt{1 - \frac{\varphi}{2c^2} - (\frac{v^2 + v\sqrt{v^2 + 2\varphi}}{2c^2})}} \quad (5)$$

Consider now a particle falling in a gravitational field such that its speed at a certain point x is v and its potintial energy is φ. According to equation (5) its mass is given by

$$m_g = m = \frac{m_0}{\sqrt{1 - \frac{\varphi}{2c^2} - (\frac{v^2 + v\sqrt{v^2 + 2\varphi}}{2c^2})}} \quad (6)$$

$$v^2 = v_0^2 - 2ax \quad (7)$$

Thus according to equations (1), (2) and (7) one gets

$$v_m^2 = \frac{v^2 + 2vv + v^2}{4}$$

$$v_m^2 = \frac{v^2 + ax + v\sqrt{v^2 + 2ax}}{2}$$

Thus equation (1) gives the inertial mass m_iis gives

$$m_i = m = \frac{m_0}{\sqrt{1 - \frac{ax}{2c^2} - (\frac{v^2 + v\sqrt{v^2 + 2ax}}{2c^2})}} \quad (8)$$

But numerically

$$ax = \varphi (9)$$

Thus

$$m_i = m = \frac{m_0}{\sqrt{1 - \frac{\varphi}{2c^2} - (\frac{v^2 + v\sqrt{v^2 + 2\varphi}}{2c^2})}} (10)$$

A direct comparison of equations (6) and (10) gives

$$m_g = m_i \quad (11)$$

This means that the inertial and gravitational masses are equivalent. Another version of equivalence principle states that the law of nature for elevator in free fall in gravitational fields takes the same form as that in free space. Now consider an elevator which is in free fall with a particle of rest mass m_0 inside it. According to equation (2)

$$v_m = \frac{v_0 + v}{2}$$

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Since the particle is at rest all times, it follows that

$$v_0 = 0 \,\&\, v = 0 \quad (12)$$

Thus

$$v_m = 0 \quad (13)$$

Thus in view of equation (1) the gravitational mass is

$$m_g = \frac{m_0}{\sqrt{1-0}} = m_0 \quad (14)$$

For a particle of rest mass inside an elevator in free space that have rest mass m_0:

$$v_0 = 0 \,\&\, v = 0 \,\&\, v_m = 0 \quad (15)$$

Thus according to equation (1) the inertial mass is given by

$$m_i = \frac{m_0}{\sqrt{1-0}} = m_0 \quad (16)$$

INERTIAL AND GRAVITATIONAL MASS IN A CURVED SPACE LORENTZ TRANSFORMATION

The equation of motion of an accelerated particle due the action of gravity field or due to the elevator acceleration a with respect to a particle in free space is given by

$$\frac{d^2x^\lambda}{dt^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt} = 0 \quad (17)$$

For slow moving particle in x-direction in a weak gravitational field, equation (17) is reduced to

$$\frac{d^2x}{dt^2} + c^2\Gamma^1_{00} = 0 \quad (18)$$

The connection in a weak field limit is given by

$$\Gamma^1_{00} = -\frac{1}{2}\mathrm{A}h_{00} \quad (19)$$

Thus the acceleration a can curve the space according to equation (18) and (19) to get

$$a = \frac{d^2x}{dt^2} = -c^2\Gamma^1_{00} = \frac{c^2}{2}\mathrm{A}h_{00} = \frac{c^2}{2}\frac{\partial h_{00}}{\partial x} \quad (20)$$

For constant acceleration

$$\int a\,dx = \frac{c^2}{2}\int dh_{00} \quad 2ax = c^2h_{00} \quad (21)$$

By using the definition of force therefore,

$$F = ma = -\mathrm{A}V = -m\mathrm{A}\varphi = -\frac{mc^2}{2}\mathrm{A}h_{00} \quad (22)$$

Thus

$$\varphi = -\frac{c^2}{2}h_{00} \,\&\, h_{00} = -\frac{2\varphi}{c^2} \quad (23)$$

Therefore, the time metric takes the form

$$g_{00} = -1 + h_{00} = -\left(1 + \frac{2\varphi}{c^2}\right) \quad (24)$$

In view of equation (21) and (24) it also takes the form

$$g_{00} = -1 + h_{00} = -1 + \frac{2ax}{c^2} = -\left(1 - \frac{2ax}{c^2}\right) \quad (25)$$

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Taking into account equation (21) and (23) one gets

$$\frac{2\varphi}{c^2} = -\frac{2ax}{c^2} \quad (26)$$

The mass in a curved space takes the form [8]

$$m = \frac{m_0}{\sqrt{-g_{00} - \frac{v^2}{c^2}}} \quad (27)$$

Using equations (27) and (24) the gravitational mass is given by

$$m_g = \frac{m_0}{\sqrt{-g_{00} - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{(1 + \frac{2\varphi}{c^2}) - \frac{v^2}{c^2}}} (28)$$

With the aid of equation (27) and (25) the inertial mass is given by

$$m_i = \frac{m_0}{\sqrt{-g_{00} - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{2ax}{c^2} - \frac{v^2}{c^2}}} (29)$$

Taking into account equations (25), (24); it follows that

$$g_{00} = -(1 + \frac{2\varphi}{c^2}) = -(1 - \frac{2ax}{c^2}) \quad (30)$$

Therefore

$$m_g = m_i \quad (31)$$

i.e. the inertial and gravitational mass are equal. For an elevator freely falling with a particle in a gravitational field, the particle is at rest with respect to him. Thus no acceleration is observed, i.e.

$$v = 0 \quad \& a = 0 \quad (32)$$

According to equation (30) and (28)

$$m_g = m_0 (33)$$

For the particle in free space, which is at rest with respect to an elevator

$$v = 0 \quad \& a = 0 \quad (34)$$

Again equation (30) and (29) gives

$$m_i = m_0 (35)$$

Thus

$$m_g = m_i \quad (36)$$

RESULTS AND DISCUSSION

Within the framework of velocity invariance and curved space Lorentz transformation, the equivalence of gravitational and inertial mass is explored. Two scenarios are offered. In the first way, the mass of a particle falling freely in a gravitational field is compared to that of a particle in free space viewed by an observer in an elevator travelling with the same acceleration as gravity. In accordance with this interpretation, the velocity-invariant Lorentz transformation demonstrates the equivalence of gravitational mass and inertial mass, as demonstrated by equation (11). This equivalence is connected to the equality of potential per unit mass and acceleration work [see equation (9)] The Lorentz transformation of curved space also demonstrates the equivalence of gravity

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and inertial mass [see equation (31)]. This is because both field and acceleration modify space, as demonstrated by equation (30). In the second form, the descending elevator with a particle in a gravitational field is compared to the elevator in free space at rest in relation to the particle. Gravitational mass equals rest mass in velocity-invariant Lorentz transformations [see equations (14) and (16)]. In the Lorentz transformation of curved space, the fact that the particle is at rest in both coordinate systems eliminates acceleration. Consequently, the space is Minkowskian, and the inertial and gravitational masses are identical to the rest mass in both frames [see equations (32) - (36)].

CONCLUSION

The velocity invariant model and the Lorentz transformation of curved space demonstrate that gravitational and inertial mass are equivalent. This equivalence relates to the equality between potential and acceleration work per unit mass.

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