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Gravitational Spin-Spin Interaction between Two Magnons and Lattice Vibration, an Explanation Using Special Relativity

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Abstract

In this work, the gravitational spin-spin force between two magnons has been discussed. It is found that the linear energy density of spin waves which has dimension of force is the spin-spin force of interaction between two magnons. Magnonic force, a new force found by us exists within a crystal lattice during the interaction between two spin waves or two magnons. The spin-spin force, interaction time as well as angular frequency and wavelength of phonon particles absorbed and created in the scattering and bonding of two magnons when their spins are parallel and anti-parallel to each other respectively has been calculated by us. We have calculated the interaction time in both S(x,y,z,t) and S^' (x^',y^',z^',t^') frame of reference using Lorentz transformation of special relativity.

Keywords: *Spin waves; Exact solutions; scattering by magnons; bound states; collisions; phonons; special relativity*

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INTRODUCTION

The gravitational spin-spin force of interaction for two mass less spinning particles is proportional to the inverse fourth power of the coordinate distance between the particles [1, 2]. Papapetrou's class of exact solution explains that the two mass less spinning particles

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possessing axially symmetric configuration which has canonical singularity representing cosmic string between the particles to balance the spin-spin force [3]. The cosmic string and linear energy density of the cosmic string has been discussed in [1, 4]. The cosmic string does not exert any gravitational field but exert a force on the particles spinning at the two ends of the string which is obviously the spin-spin force which is repulsive if the spins are parallel and attractive if the spins and anti-parallel [1-7]. There are several papers [8-15] explaining pairs of massive spinning particles which is based upon the exact stationary solutions of Einstein's equations.

A magnon is considered as a quasiparticle which is collective excitation of the electron's spin structure in a crystal lattice. In quantum mechanics, a magnon is a quantized spin wave. A magnon is a spin 1 quasiparticle which obeys the boson behaviour [16, 17]. Magnons carry a fixed amount of energy and lattice momentum. Nano-sized spin-wave devices are used in order to produce spin waves [18-20]. Generally, two cases arise in the interaction between two magnons. In first case, for parallel spins the magnons scatter against each other. In second case, for anti-parallel spins the magnons form bound states [17]. There are several papers on spin waves and magnons, a few of them are [21-40]. In the spin-spin interaction between two magnons there is lattice vibration which is quantized as phonon particles having certain energy and momentum with spin 0 [41-44].

In this work, first we discuss about the energy and momentum of quantized spin wave i.e. quasiparticle magnon. The spin waves are quantized as magnons which is similar to the quantization of electromagnetic waves and elastic waves as photons and phonons respectively. First, we find out the linear energy density of the spin waves defined as $\frac{Energy}{WWavelengt\ h}$ which is the spin-spin force between two magnons in the interaction between two spin waves travelling within crystal lattice. After this, we have find the Magnonic force F_M which is a new force calculated by us acting at about 100 μm within a crystal lattice during the interaction between two spin waves or two magnons. Then we have find out the gravitational spin-spin force of interaction between two magnons. In crystal lattice, if there is flip in spin angular momentum of electron from $\hbar/2$ to $-\hbar/2$, a magnon is emitted. Two spin waves coming from opposite direction will collide and hence there will be

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collision between two magnons, in which there is a force of interaction between them which may be repulsive or attractive depending upon their spin. If the initial spin angular momentum of the first magnon is $\hbar$ and that of the second magnon also $\hbar$, there will be force of repulsion. If the initial spin angular momentum of first magnon is $\hbar$ and that of second magnon is -$\hbar$, there will be force of attraction and two magnons will be bonded together. Then we calculate gravitational spin-spin force of interaction between the two magnons. Finally, we have found out the time of interaction in the interaction between two magnons in both $S(x, y, z, t)$ frame of reference used for magnons which is Centre of mass frame of reference and S^' (x^',y^',z^',t^') frame of reference is used for magnon itself moving with velocity v. Here we have used the principle of collision theory in which impulse i.e. force of interaction multiplied by time of contact or interaction is equal to the change of momentum of the colliding bodies. Finally, we calculate the energy and the momentum of the phonon particles where + sign indicate the creation of phonon particles and - sign indicate the absorption of phonon particles [41,44], having ω_P and λ_P as angular frequency and wavelength of the phonon particles respectively.

This paper is organized as follows: In section 2, we discuss about the energy and momentum of quantized spin waves magnons, quantized elastic waves phonons and special relativity. In section 3, we have found out Magnonic force which exists at the level of about 100 μm during the interaction between spin waves or magnons. In sections 4 and 5, we calculate the time of interaction considering the gravitational spin-spin interaction between two scattered magnons and two bonded magnons as well as displacement of magnons in both $S(x, y, z, t)$ and $S'(x', y', z', t')$ frame of references respectively. In section 6, we have gone through results and discussions. Finally, we present our conclusions in section (7).

Energy and momentum of magnons and phonons; and special relativity

Since, magnons are quantized spin waves which are created in crystal lattice, if there is flip in spin of the electron such that its spin angular momentum is changed from $\hbar/2$ to $-\hbar/2$ or from$-\hbar/2$ to $\hbar/2$ [16]. This phenomena takes place in insulators, semiconductors and metals when the interaction between spins of electrons occur. Nano-sized spin-wave devices are used for this purpose [18-20]. The energy of spin waves are in μeV. The wavelength

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of spin waves are in the range of $200 - 300\ \mu m$ and their group velocity corresponds to $10^5\ m/s$ [39,40]. The quantized spin waves, magnons are massless quasiparticles we can define the wavelength of the spin waves in terms of the Compton wavelength of the electron and time period of the spin waves can be defined in terms of the time on Compton scale. Also, the energy of the spin wave can be defined in terms of the rest energy of the electron as [45]:

$$m_e c^2 = 0.51 \times 10^6 eV = 0.51\ MeV. \quad (1)$$

The Compton wavelength of the electron is defined as,

$$\lambda_C = \frac{\hbar}{m_e c} = 3.872 \times 10^{-13} m. \quad (2)$$

On the Compton scale, time is given by,

$$T_C = \frac{\hbar}{m_e c^2} = 1.3 \times 10^{-21} \text{s}. \quad (3)$$

So, energy on the Compton scale, i.e. Compton energy of electron can be defined as

$$E_C = h\nu_C = \frac{h}{T_C} = 2\pi m_e c^2 = 3.204 MeV. \quad (4)$$

The spin waves are generated due to spin of the electron which is quantized as magnons. We can define the wavelength of the spin waves in terms of the Compton wavelength of the electron using equation (2) as

$$\lambda = q\lambda_C \times 10^7 = \frac{q\hbar}{m_e c} \times 10^7, \quad (5)$$

where, $q \in]52, 75]$ can be any integer or fraction in equation (5).

The energy of the spin waves which are in μeV can be defined in terms of the Compton energy of the electron can be given by

$$E = \hbar\omega = 2\pi a m_e c^2 \times 10^{-12}, \quad (6)$$

where, $a \in]1, 100]$ can be any integer or fraction in equation (6). From equations (4) and (6), we will get

$$\frac{E}{E_C} = \frac{h\nu\nu}{h\nu_C} = \frac{T_C}{T}. \quad (7)$$

From equation (7), we can find out the time period of spin waves as

$$T = \frac{E_C}{E} T_C = \frac{1.3 \times 10^{-9} s}{a}, \quad (8)$$

where, $a \in]1, 100]$ can be any integer or fraction in equation (8). The angular frequency of the spin waves can be defined using equation (8) as

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$$\omega = \frac{2\pi}{T} = \frac{2\pi a}{1.3\times10^{-9}s} = 1.6a\ GHz, \quad (9)$$

where, $a \in]1, 100]$ can be any integer or fraction in equation (9). The momentum of the quantized spin wave magnon quasiparticle is given below as:

$$p = \frac{h}{\lambda} = \frac{2\pi m_e c}{q} \times 10^{-7}, \quad (10)$$

where, $q \in]52, 75]$ can be any integer or fraction in equation (10). The group velocity of the spin waves are found to be of the order of $10^5 m/s$ which is also the velocity of each quantum of spin wave the magnon. In equation (10), q determines the momentum of the magnon. If the two spin waves coming from opposite direction collide, the two colliding magnons may scatter or may get bonded with each other. The collision of two magnons is completely inelastic collision. Hence we can obtain force of interaction (linear energy density of spin waves) multiplied by interaction time i.e impulse between two interacting magnons as the change in momentum of magnon and is defined as:

$$F\Delta t = \Delta p. \quad (11)$$

Equation (11) is based upon principle of collision theory. If a ball falls on the surface of the earth, then the impulse i.e. the force of interaction (gravitational force) between ball and earth's surface multiplied by the time of contact or time of interaction is equal to change in momentum of the ball. Here, we have used this principle to explain the interaction between two magnons where force of interaction is the spin-spin force which when multipled by the time of interaction between the two magnons gives change in momenta of the magnons.

Using Heisenberg's Uncertainty Principle i.e.

$$\Delta E \Delta t \geq \hbar/2, \quad (12)$$

we can find out the energy difference during the collision between two magnons using the value of Δt in equation (12). Again using the Heisenberg's Uncertainty Principle i.e.

$$\Delta x \Delta p \geq \hbar/2, \quad (13)$$

we can find out the displacement of magnons in this collision using the value of Δp in equation (12). The interacting distance between two magnons can be calculated using equation (5) which corresponds to the wavelength of the spin waves.

There is lattice vibration in the spin-spin interaction between the two magnons which is quantized as phonon particles

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having certain energy and momentum with spin 0. [41-44]. The energy and the momentum of the phonon particle is given as

$$E_P = \pm\hbar\omega_P, \quad (14)$$

$$p_P = \pm\frac{h}{\lambda_P}, \quad (15)$$

where ω_P and λ_P are angular frequency and wavelength of the phonon particles respectively. In equations (14) and (15) + sign indicate the creation of phonon particles and - sign indicate the absorption of phonon particles [41, 44].

In special relativity, the time in frames of references $S(x, y, z, t)$ and $S'(x', y', z', t')$ are related through Lorentz transformation as [46,47]:

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}. \quad (16)$$

Also in special relativity, the displacement in frames of references $S(x, y, z, t)$ and $S'(x', y', z', t')$ are related through Lorentz transformation as [46, 47]:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad (17)$$

$$y' = y, \quad (18)$$

$$z' = z. \quad (19)$$

Magnonic Force

The energy, wavelength and group velocity of spin waves corresponds to $\mu eV, 200 - 300\ \mu m$ and $10^5\ m/s$ respectively [18-20, 39, 40]. The quantized spin waves, magnons are massless quasiparticles. The spin waves travelling within the lattice may interact with each other. Hence, the quantized spin waves magnons will interact with each other which may scatter with different energies what they initially have or may get coupled with each other. The linear energy density of the spin waves can be defined as $\frac{Energy}{WWavelengt\ h}$. Considering the energy of the spin wave as 1 μeV and the wavelength of the spin wave 100 μm, the linear energy density calculated is $0.01 eV/m$. The linear energy density of spin waves has dimension of force is the force of interaction between two magnons which can be the spin-spin force between two magnons. The value of the spin-spin force between two magnons as linear energy density of spin waves is calculated as $F = 1.6 \times 10^{-21} N$. We can compare our result with Papapetrou's class of exact solution explaining the two massless spinning particles possessing axially symmetric configuration which has canonical singularity representing cosmic string between the particles to balance the spin-spin force is also the negative of the

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linear energy density of the cosmic string [1,3,4].The metric for axially symmetric space time is given as [1]:

$$ds^2 = -f^{-1}e^{vv}(dz^2 + dr^2) - lddd^2 - 2ndddt + fdt^2 \quad (20)$$

in which f, vv, l, n are functions of r and z. The conical singularity in the equation (20) is given by,

$$ds^2 = -f_0^{-1}(dZ^2 + d\rho^2 + \rho^2 e^{-2C}ddd^2) + f_0 dt^2, \quad (21)$$

where, $\rho = re^C, Z = ze^C, f_0 = \lim_{r\to 0} f$, $|z| < b$. The conical singularity in equation (15) represents the cosmic string. The equation of state of cosmic string is given by $p + \lambda' = 0$, where $\lambda' = \frac{1}{4}(1 - e^{-C})$ is linear energy density of the cosmic string [1,4]. Actually, we have $\lambda' = \frac{c^4}{4G}(1 - e^{-C})$, where $c = G = 1$ is taken as the cosmic string exists at the Planck scale. But we have gravitational spin-spin force from Planck scale to atomic and molecular level. Not only is this there existence of spin-spin force at cosmological levels like the spin-spin interaction between two black holes, two neutron stars and a black hole and a neutron star, etc.

The measure of the spin-spin force is given below as [1-7]:

$$p' = -\lambda' = -\frac{1}{4}(1 - e^{-C}) = -\frac{c^4}{4G}(1 - e^{-C}), \text{ where } c = G = 1. \quad (22)$$

The force exerted on the particles spinning at the two ends of the cosmic string is as given in equation (22). In equation (22) we have taken, $C = \frac{3Gh_1h_2}{2c^2b^4F_M}$ a dimensionless quantity and ratio within exponential term when spin-spin force between two massless spinning particles are considered. We have taken F_M in the denominator of the constant C in order to make it dimensionless since we are discussing about the spin-spin force between two magnons which is taking place at about 100 μm and which can be calculated as:

$$p' = -\frac{c^4}{4G}(1 - e^{-C}) = -\frac{c^4}{4G}(1 - e{-3Gh1h22c2b4FM}. \quad (23)$$

In equations (22) and (23), p' is defined as spin-spin force which is also the negative of the linear energy density of the cosmic string. C is a ratio and a dimensionless quantity within exponential term in equations (22) and (23).

Thus we can obtain spin-spin force between the two interacting magnons

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using equation (23) taking $h_1 = \hbar$ as spin angular momentum of the first magnon and $h_2 = -\hbar$ as the spin angular momentum of the second magnon. Hence, we have to consider$b \sim \lambda_M = 100\ \mu m$, as the distance of interaction between two magnons, the simplified expression for spin-spin force during interaction of spin waves is obtained from equation (23) as

$$F_s = \frac{3c^2\hbar^2}{8b^4 F_M} \quad (24)$$

Now we can use the value of the force of interaction between two magnons calculated as $F = 1.6 \times 10^{-21} N$ which is also the linear energy density of the spin waves, in order to find the value of F_M. Considering the force of interaction between two magnons as the gravitational spin-spin force of interaction between two magnons and putting the value of $F_s = F = 1.6 \times 10^{-21} N$ in equation (24), we will obtain the value of Magnonic force F_M as

$$F_M = \frac{3c^2\hbar^2}{8b^4(1.6\times10^{-21}N)} = 2.34 \times 10^{-15} N. \quad (25)$$

Magnonic force F_M is a new force calculated by us which acts at about $100\ \mu m$ within a crystal lattice during the interaction between two spin waves or two magnons.

Gravitational spin-spin interaction between two scattered magnons and Lattice vibration

Thus, the spin-spin force between two scattered magnons can be given using equation (23) as:

$$F_s = -\frac{c^4}{4G}\left(1 - e^{-\frac{3G\hbar^2}{2c^2 b^4 F_M}}\right), \quad (26)$$

where, $h_1 = \hbar$ is spin angular momentum of the first magnon and $h_2 = \hbar$ is the spin angular momentum of the second magnon. Here, we have considered $b = \frac{q\hbar}{m_e c} \times 10^7 e^{-\frac{h_1}{h_2}} = \frac{q\hbar}{m_e c} \times 10^7 e^{-1}$ as the minimum distance of interaction between two scattered magnons since the force is repulsive where, $q \in]52, 75]$ can be any integer or fraction. From equation (26), it is clear that if the spin-spin interaction force is repulsive, the spins of two magnons will be parallel. And hence, the spin angular momentum of both the magnons will be of same sign. We have to consider the minimum interacting distance which is dependent upon the ratio of the spins of two interacting particles. Putting the value of $b = 200\ \mu m$ in equation (26), we will get the spin-spin force of

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interaction between two scattered magnons. The spin-spin force between two scattered magnons calculated using equation (26) is $F_s = -5.03 \times 10^{-21} N.$ – ve sign shows that there is repulsive force of interaction between two scattered magnons. In our calculations, we have taken the values $G = 6.674 \times 10^{-11} m^3 Kg^{-1} s^{-2}, \hbar = 1.054 \times 10^{-34} J.s = 6.582 \times 10^{-16} eV.s, m_e = 9.11 \times 10^{-31} Kg = 0.51\, MeV, r_e = 2.82 \times 10^{-15} m, F_M = 2.34 \times 10^{-15} N$.

The interaction time between two magnons during the scattering of two magnons in the S(x,y,z,t) frame of reference can be calculated using equation (11). We can write down the expression of the change in momentum in S(x,y,z,t) frame of reference as,

$$\Delta p = -\frac{h}{\lambda_1} + \frac{h}{\lambda_2} - \frac{h}{\lambda_3} + \frac{h}{\lambda_4}, \quad (27)$$

where, Δp should be – ve in order to satisfy equation (11). From equation (27), the value of Δp calculated for $\lambda_1 = \lambda = 200\, \mu m$ $\lambda_2 = 1.2\lambda$, $\lambda_3 = 1.5\lambda$ and $\lambda_4 = 1.4\lambda$ in $S(x, y, z, t)$ frame of reference is

$$\Delta p = -0.12 \frac{h}{\lambda} \approx -4.0 \times 10^{-31`} Kg. \frac{m}{s} \quad (28)$$

In equation (28), – ve value of Δp shows that spin-spin force is also – ve and hence repulsive. Using Lorentz transformation of special relativity, the interaction time in the scattering of two magnons in the $S'(x', y', z', t')$ frame of reference will be given by [46,47]:

$$\Delta t' = \frac{\Delta t - \mathrm{v}\Delta x/c^2}{\sqrt{-\mathrm{v}^2/c^2}}. \quad (29)$$

In equation (29), v is the velocity of magnon and Δx is displacement of the magnon after scattering. Using Lorentz transformation of special relativity, the displacement in the scattering of two magnons in $S'(x', y', z', t')$ frame of reference will be given by [46,47]:

$$\Delta x' = \frac{\Delta x - \mathrm{v}\Delta t}{\sqrt{-\mathrm{v}^2/c^2}}. \quad (30)$$

There is lattice vibration in the spin-spin interaction between two scattered magnons which is quantized as phonon particles having spin 0 [41-44]. From equations (11), (26) and (28), we observe that the product of spin-spin force and interaction time is negative as a result of which the change in momentum is also negative. So, there will be absorption of phonon particles in the scattering of two interacting magnons. Hence, the energy and the momentum of the phonon particle

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absorbed can be given using equations (14) and (15) as

$$E_P = \Delta E = -\hbar\omega_P, \quad (31)$$

$$p_P = \Delta p = -\frac{h}{\lambda_P}, \quad (32)$$

where ω_P and λ_P are angular frequency and wavelength of the phonon particles respectively absorbed during the scattering of two magnons. In equations (31) and (32), ΔE and Δp are energy difference and change in momentum during the scattering of two magnons when they interact with each other having parallel spins.

Gravitational spin-spin interaction between two bonded magnons and Lattice vibration

The gravitational spin-spin force between two bonded magnons is given using equation (23) as:

$$F_s = -\frac{c^4}{4G}\left(1 - e^{\frac{3G\hbar^2}{2c^2b^4F_M}}\right), \quad (33)$$

where, $h_1 = \hbar$ is spin angular momentum of the first magnon and $h_2 = -\hbar$ is the spin angular momentum of the second magnon. Hence, we have to consider $b = \frac{q\hbar}{m_e c} \times 10^7 e^{-\frac{h_1}{h_2}} = \frac{q\hbar}{m_e c} \times 10^7 e$ as the maximum distance of interaction between two interacting magnons. If the spin-spin interaction force is attractive, the spins of two magnons will be anti-parallel and hence the spin angular momentum of both the magnons will be of opposite sign. We have to consider the maximum interacting distance which is dependent upon the ratio of the spin of two interacting particles. Putting the value of $b = 300\ \mu m$ in equation (33), we will get the spin-spin force of interaction between two bonded magnons. The spin-spin force between two bonded magnons calculated using equation (33) is $F_s = 3.62 \times 10^{-25} N$. $+ve$ sign shows that there is attractive force of interaction between two bonded magnons.

The interaction time between two magnons during the bonding of two magnons in the $S(x, y, z, t)$ frame of reference can be calculated using equation (11). In this case Δp is $+ve$ since the spin-spin force is also $+ve$ and hence attractive. In the coupling of two magnons with anti-parallel spins they will be vanished and will lead to the creation of one quantum of phonon particle having certain energy and momentum. We can calculate the change in momentum in $S(x, y, z, t)$ frame of reference from equation (27). The value of Δp calculated for $\lambda_1 = \lambda = 300\ \mu m$, $\lambda_2 = 0.2\lambda$, $\lambda_3 = \infty$ and $\lambda_4 = \infty$ in

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$S(x, y, z, t)$ frame of reference from equation (27) is

$$\Delta p = 4\frac{h}{\lambda} = 8.8344 \times 10^{-30`} Kgm/s. \quad (34)$$

Using Lorentz transformation of special relativity, the interaction time in the bonding of two magnons in the $S'(x', y', z', t')$ frame of reference will be given by equation (29), in which v is the velocity of magnon and Δx is displacement of the magnon during this process. Also, the displacement in the bonding of two magnons in $S'(x', y', z', t')$ frame of reference will be given by equation (30).

There is lattice vibration in the spin-spin interaction between two bonded magnons which is quantized as phonon particles having spin 0 [41-44]. From equations (11), (33) and (34), we observe that the product of spin-spin force and interaction time is positive as a result of which the change in momentum is also positive. So, there will be creation of phonon particles in the bonding of two interacting magnons. Hence, the energy and the momentum of the phonon particle created can be given using equations (14) and (15) as

$$E_P = \Delta E = +\hbar\omega_P, \quad (35)$$

$$p_P = \Delta p = +\frac{h}{\lambda_P}, \quad (36)$$

where ω_P and λ_P are angular frequency and wavelength of the phonon particles created during the bonding between two magnons repectively. In equations (35) and (36), ΔE and Δp are energy difference and change in momentum during the bonding between two magnons when they interact with each other having anti-parallel spins and vanishes after some time to create a phonon particle.

RESULTS AND DISCUSSIONS

Considering linear energy density of spin waves as $0.01eV/m$, the value of the spin-spin force between two interacting magnons is calculated as $1.6 \times 10^{-21}N$.. The value of the Magnonic force which is a new force calculated by us is $F_M = 2.34 \times 10^{-15`}N$. Using equations (11), (26) and (28) the time of interaction in the scattering of two magnons in $S(x, y, z, t)$ frame of reference is calculated as $\Delta t = 8.0625 \times 10^{-11}s$. The value of energy difference ΔE calculated in this case in $S(x, y, z, t)$ frame of reference using equation (12) as $\Delta E = 4.08\ \mu eV$. The value of displacement in this case can be calculated using equations (13) and (28) as $\Delta x \approx -132\ \mu m$. The energy and the momentum of the phonon particle

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absorbed in this case can be given from equations (31), (28) and (32) as $E_P = -4.08\,\mu eV$ and $p_P = -4.0 \times 10^{-31}$`$Kg.\ m/s$ respectively. Hence, angular frequency and wavelength of the phonon particles absorbed during the scattering of two magnons can be calculated as ω_$\omega_P = 6.2\,GHz$ and $\lambda_P = 1657\,\mu m$. The interaction time in the scattering of two magnons in $S'(x', y', z', t')$ frame of reference can be calculated using equation (29) as $\Delta t' \approx 8.06248547 \times 10^{-11}$s. The value of displacement in this case can be calculated using equation (30) as $\Delta x' \approx -140.0625024\,\mu m.$ Using equations (11), (33) and (34) the time of interaction in the bonding between two magnons in $S(x, y, z, t)$ frame of reference is calculated as $\Delta t = 8.125 \times 10^{-7} s.$ The value of energy difference ΔE calculated in this case in $S(x, y, z, t)$ frame of reference using equation (12) as $\Delta E = 0.405\ neV$. The value of displacement in this case can be calculated using equations (13) and (34) as$\Delta x \approx 533\ \mu m.$ The energy and the momentum of the phonon particle created in this case can be given from equations (34), (35) and (36) as $E_P = 0.405\,neV$ and $p_P = 8.8344 \times 10^{-30}$`$Kg.\ m/s$ respectively. Hence, angular frequency and wavelength of the phonon particles created during the scattering of two magnons can be can be calculated as $\omega_P = 612\ GHz$ and $\lambda_P = 75\ \mu m$. The interaction time in the scattering of two magnons in $S'(x', y', z', t')$ frame of reference can be calculated using equation (29) as $\Delta t' \approx 8.125000132 \times 10^{-7}$s. The value of displacement in this case can be calculated using equation (30) as $\Delta x' \approx -8.071700137\ cm.$ Here we have used the velocity of magnon v = 10^5m/s, Lorentz facor $\gamma\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1.000000017.$

CONCLUSIONS

In our work, we have got two new things. The first one is that the linear energy density of the spin waves is the spin-spin force between two magnons and the second one is the Magnonic force F_M. The spin waves which can be quantized as magnons are massless quasiparticles travelling within the lattice may interact with each other. The result would be that the two interacting magnons will be either scattered if they have parallel spins with different energies what they initially have or may get coupled with each other and vanishes out if they have anti-parallel spins. After comparing our result with Papapetrou's class of exact solution in which two massless spinning particles

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possess axially symmetric configuration having canonical singularity representing cosmic string between the particles to balance the spin-spin force which is also the negative of the linear energy density of the cosmic string [1,3,4], we have come to the conclusion that .the linear energy density of the spin waves which can be defined as Energy/Wavelength has dimension of force is also the spin-spin force of interaction between two magnons as in the case of cosmic string.

Using the concept of gravitational spin-spin interaction between two magnons, we can find out the time of interaction in the scattering of two magnons in both S(x,y,z,t) and S^' (x^',y^',z^',t^') frames of references. We have considered spins of two scattered magnons to be parallel to each other in the interaction of two magnons since the force should be repulsive liable for scattering of magnons.

Similarly, for the case of bonding between two magnons, where we have considered spins of two magnons to be anti-parallel to each other and the gravitational spin-spin force to be attractive so that the two magnons make bond with each other. We have also found out the time of interaction in the bonding of two interacting magnons for antiparallel spin states in both the S(x,y,z,t) and S^' (x^',y^',z^',t^') frames of references.

Finally, we have come to the conclusion that there is lattice vibration in the spin-spin interaction between two scattered magnons which is quantized as phonon particles having spin 0 [41-44]. There will be absorption of phonon particles in the scattering of two interacting magnons having parallel spins. In the spin-spin interaction between two magnons having anti-parallel spins will couple with each other and will be vanished out leading to the creation of phonon particles.

The biggest achievement of our work is the searching out of the Magnonic force existing at about 100 μm within a crystal lattice during the interaction between two spin waves or spin-spin interaction between two magnons is totally a new force and its value calculated by us is F_M=2.34× 〖10〗 ^(-15`) N. Hope our results will be proved both theoretically and experimentally in the near future.

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