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Topology Preserving Toolpath Optimization for Complex Freeform Surfaces

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Abstract

The increasing demand for high-precision manufacturing of complex freeform surfaces in aerospace, automotive, and biomedical industries has necessitated the development of advanced toolpath optimization methods. Traditional toolpath generation often compromises surface quality and machining efficiency due to topological inconsistencies and redundant tool motions. This paper presents a comprehensive review of ***topology preserving toolpath optimization techniques****, which maintain the geometric and topological integrity of freeform surfaces while minimizing machining time, tool wear, and errors. The study discusses key algorithms, including iso-parametric, iso-planar, and adaptive slicing strategies, along with topology-aware path planning methods. Case studies and simulation results demonstrate significant improvements in machining accuracy and efficiency. Challenges in implementation and future research directions are also highlighted.*

Keywords: *Toolpath optimization, freeform surfaces, topology preservation, CNC machining, adaptive slicing, surface integrity, path planning.*

INTRODUCTION

Manufacturing of **complex freeform surfaces** has become critical in precision engineering, especially in fields like aerospace, automotive, biomedical implants, and die-making. Freeform surfaces are characterized by intricate geometries with non-uniform curvature and irregular

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topology, making conventional machining strategies inefficient and error-prone. Traditional CNC toolpaths, such as simple zig-zag or raster patterns, often result in redundant passes, surface inaccuracies, and collisions in areas of high curvature.

Topology preserving toolpath optimization (TPTO) addresses these challenges by generating machining paths that adhere to the surface's intrinsic topological structure. The main objectives of TPTO include:

1. **Maintaining geometric integrity** – ensuring the toolpath does not distort the surface features.
2. **Minimizing machining time** – reducing unnecessary tool movements.
3. **Reducing tool wear** – optimizing feed rate and cutting depth based on surface **curvature**.
4. **Ensuring process safety** – preventing collisions and maintaining stable tool engagement.

This paper reviews recent methodologies, tools, and algorithms for TPTO, discusses comparative studies, and identifies future research directions.

FREEFORM SURFACES AND TOOLPATH CHALLENGES

The machining of **freeform surfaces** has become increasingly important in modern manufacturing due to the rising demand for complex components with high precision and surface quality. Freeform surfaces are distinct from standard geometric shapes such as planes, spheres, cylinders, or simple extrusions because they do not follow a simple analytical expression. Instead, these surfaces are defined by smoothly varying curves and topologies that allow designers to create highly complex and customized shapes.

1. Freeform Surfaces

Freeform surfaces can be described as smooth, continuous surfaces whose curvature changes along their length and width. Unlike simple surfaces, they often contain multiple concavities, convexities, and undulations that require careful consideration during the machining process. The ability to design and manufacture these surfaces is critical in several industries:

- **Automotive body panels:** Modern car exteriors often incorporate freeform surfaces to improve aerodynamics, aesthetics, and performance. Each panel has unique curvature that

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varies in three dimensions.

- **Turbine blades:** Gas turbines and jet engines use blades with complex aerodynamic profiles. Slight deviations in curvature can reduce efficiency or cause mechanical failure.

- **Orthopedic implants:** Customized implants, such as hip or knee replacements, must conform to patient-specific anatomical shapes, which are inherently freeform and highly irregular.

- **Mold and die surfaces:** Injection molds and forging dies require precise surface geometries to produce parts with tight tolerances and high-quality finishes.

Mathematical Representation:

Freeform surfaces are typically represented using **Non-Uniform Rational B-Splines (NURBS)** or **Bezier surfaces**. These mathematical models allow for precise control of surface curvature through control points and weighting functions. NURBS surfaces are widely preferred because they can represent both standard analytical shapes and complex freeform geometries accurately.

The main characteristic of freeform surfaces is **non-uniform curvature**, meaning the rate of curvature change varies across the surface. This irregularity presents a significant challenge in toolpath generation because a uniform feed rate or step-over distance may result in surface inaccuracies, tool deflection, or even collisions. Additionally, topological features such as holes, depressions, or protrusions complicate path planning further.

2. Challenges in Toolpath Planning

Generating an efficient and accurate toolpath for freeform surfaces involves several technical challenges:

1. **Surface Undercutting:**

When machining concave regions or sharp depressions, the cutting tool must be carefully oriented to avoid gouging the surface. Undercutting occurs when the tool penetrates beyond the intended surface geometry, causing defects that are difficult to correct. For example, machining a turbine blade root with high concavity can easily result in gouging if tool

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orientation is not optimized.

2. **Redundant Machining:**

Conventional toolpath strategies, such as linear zig-zag or raster paths, often result in repeated passes over certain regions. Redundant machining not only increases **cycle time** but also accelerates **tool wear** and reduces process efficiency. This is particularly problematic for large freeform surfaces like automotive hoods or molds.

3. **Curvature Variation:**

Freeform surfaces typically include regions of both low and high curvature. High curvature areas require slower feed rates and smaller step-over distances to maintain accuracy, while flat regions can tolerate faster machining. Failure to adapt the toolpath to curvature changes can lead to **poor surface finish** and dimensional inaccuracies. For instance, a hip implant surface with varying curvature demands adaptive tool engagement to maintain smooth contours without micro-gouges.

4. **Topological Complexity:**

Freeform surfaces often contain multiple features such as holes, pockets, bumps, and depressions. Each of these features necessitates **localized adaptive strategies** in toolpath generation. A uniform toolpath approach may miss small features or cause collisions in intricate regions. Machining a complex mold insert with several pockets and fillets, for example, requires advanced planning algorithms that can navigate around obstacles while preserving the surface topology.

5. **Machining Dynamics:**

Additional challenges include tool deflection, vibration, and thermal deformation, all of which are amplified in freeform surface machining due to non-uniform cutting conditions. These factors must be considered when planning toolpaths to ensure that both geometric accuracy and surface quality are maintained.

In summary, toolpath planning for freeform surfaces is not merely a question of connecting points on a surface but involves **balancing efficiency, accuracy, and tool safety** while respecting the intricate topological features of the part. This necessitates **advanced algorithms**,

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such as curvature-based adaptive paths, topology-preserving methods, and multi-axis strategies, to achieve the desired outcomes.

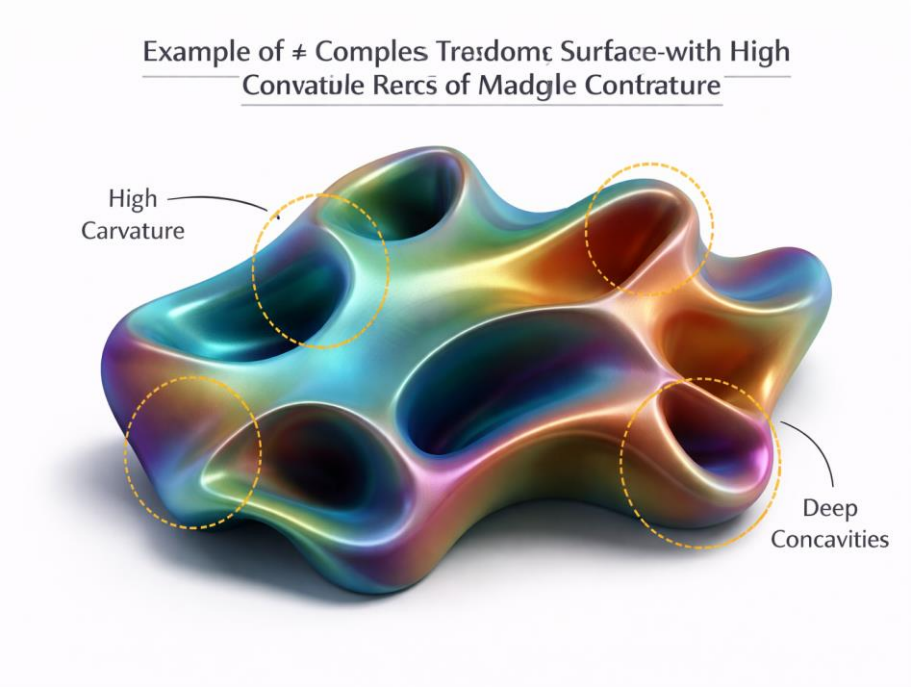


Figure 1: Example of a complex freeform surface with high curvature areas and multiple concavities

TOOLPATH OPTIMIZATION TECHNIQUES

Efficient and accurate toolpath generation is a critical factor in machining complex freeform surfaces. The goal of toolpath optimization is to minimize machining time, reduce tool wear, and maintain surface integrity, while respecting the geometric and topological features of the surface. Over the years, several strategies have been developed to achieve these objectives, ranging from simple iso-parametric methods to advanced adaptive and topology-preserving algorithms. This section discusses the key techniques used in toolpath optimization, highlighting their principles, advantages, and limitations.

1. Iso-Parametric Toolpaths

Iso-parametric toolpath generation is one of the earliest and simplest approaches used for freeform surface machining. In this method, the surface is represented in its parametric form using coordinates **(u, v)**, where u and v define the parametric directions along the surface. Toolpaths are generated by moving the cutting tool along constant parametric intervals, either

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in the u-direction, v-direction, or a combination of both. Essentially, the tool follows the natural parametric grid of the surface.

Principle:

- Divide the parametric space (u, v) of the surface into uniform increments.
- Generate a sequence of points corresponding to each increment.
- Connect these points to form continuous toolpaths for the cutting tool.

Example: Consider machining an automotive hood designed as a NURBS surface. By sampling points along constant u and v intervals, iso-parametric paths allow the tool to follow the underlying surface structure.

Advantages of Iso-Parametric Toolpaths:

1. Ease of Implementation:

Iso-parametric methods are straightforward and do not require complex calculations. Most CAD/CAM software supports parametric surface representation, making this method compatible with standard manufacturing workflows.

2. Direct Integration with CAD Models:

Since the toolpath follows the parametric representation of the surface, there is minimal preprocessing required, and the path can be directly exported to CNC machines.

3. Consistent Path Planning:

For surfaces with relatively uniform curvature, iso-parametric paths provide a structured and predictable tool movement, simplifying machine control.

Limitations of Iso-Parametric Toolpaths:

1. **Uneven Surface Finish:**

Freeform surfaces often exhibit **variable curvature**. Iso-parametric paths, based on uniform parametric intervals, result in **uneven step-over distances** in 3D space. In areas of high curvature, points are closer together, leading to excessive tool overlap, whereas in flatter areas, points may be farther apart, causing scallop formation. This results in a non-uniform surface finish.

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2. **Redundant Passes in Flat Regions:**

Because iso-parametric paths follow the surface grid uniformly, they may include unnecessary passes over regions that are nearly flat. This redundancy increases machining time and tool wear without improving surface quality.

3. **Limited Adaptability:**

Iso-parametric methods do not inherently account for **topological features**, such as holes, fillets, or concavities. As a result, manual intervention or additional path correction is often required to prevent tool collisions or gouging.

4. **Inefficient for Multi-Axis Machining:**

In 5-axis or multi-axis CNC setups, iso-parametric paths may fail to optimize tool orientation, leading to potential undercuts and inefficient tool engagement.

Table 1: Comparison of Iso-Parametric and Iso-Planar Toolpaths

Toolpath Type	Advantages	Disadvantages	Applications
Iso-Parametric	Simple, easy CAD integration	Surface errors in high curvature	Automotive panels, dies
Iso-Planar	Uniform step-over, better surface finish	Difficult in steep slopes	Aerospace blades, molds

2. Iso-Planar Toolpaths

Iso-planar toolpath strategies are widely used in both subtractive manufacturing (milling) and additive manufacturing processes. Unlike iso-parametric methods, which follow the parametric coordinates of a surface, iso-planar techniques generate **toolpaths along a series of horizontal planes**, slicing the 3D surface at uniform or variable heights. The intersection of each plane with the surface defines a contour, which is then traced by the cutting tool.

Principle:

- The surface is divided into **parallel planes**, often along the Z-axis.
- Each plane intersects the surface, producing **iso-height contours**.

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- The cutting tool follows these contours sequentially, layer by layer, until the complete surface is machined.

Advantages of Iso-Planar Toolpaths:

1. **Uniform Step-Over and Surface Finish:**

Since the tool moves along planar contours, the distance between successive passes is more uniform in 3D space, which improves surface smoothness compared to iso-parametric methods.

2. **Simplicity and Predictability:**

Iso-planar paths are easier to visualize and control, making them suitable for standard milling and 3-axis CNC machines.

3. **Compatibility with Additive Manufacturing:**

In 3D printing, iso-planar slicing is the standard approach for layer-by-layer deposition, making this method versatile across both subtractive and additive processes.

Limitations and Challenges:

1. **Ignoring Complex Topology:**

Iso-planar slicing assumes a relatively simple surface intersection with horizontal planes. For surfaces with **holes, pockets, or steep concavities**, iso-planar paths may miss critical features or require additional passes to cover hidden regions. For example, machining a mold insert with internal pockets may result in uncut areas if the contours do not intersect the cavity correctly.

2. **Over-Machining in Thin Features:**

In areas where the surface thickness is small, horizontal slicing may cause the tool to engage excessively with the material, leading to overcutting or thinning of the part. Thin ribs and fillets are particularly susceptible to this problem.

3. **Limited Adaptation to Curvature:**

Iso-planar paths generally maintain a fixed step-over, which may not adequately account for **high-curvature regions**. This can result in scallops or poor surface finish in areas with rapid geometry changes.

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Example:

In the case of an aerospace turbine blade, iso-planar paths at uniform heights will accurately machine the leading and trailing edges but may **miss or overcut concave cooling channels** within the blade. Figure 2 illustrates the concept of iso-planar slicing and highlights potential gaps or overcuts in complex topologies.

3. Adaptive Toolpath Generation

Adaptive toolpath generation represents a significant advancement over traditional iso-parametric and iso-planar methods. These methods dynamically adjust the toolpath based on **surface curvature, feature complexity, and topological constraints** to optimize machining efficiency and surface quality.

Principle:

- The toolpath parameters, such as **step-over, feed rate, and tool orientation**, are continuously adapted according to the geometry of the surface.
- High curvature regions are machined with finer passes and slower feed rates to preserve accuracy, while flatter regions allow faster machining.

Key Techniques in Adaptive Toolpath Generation:

1. **Curvature-Based Step-Over Adjustment:**

- The surface is analyzed for curvature using mathematical models (e.g., Gaussian curvature).
- Step-over distance is **smaller in highly curved regions** to avoid scallops or undercuts.
- Flat or low-curvature regions use **larger step-over distances** to reduce machining time. *Example:* A hip implant surface with varying curvature can be machined more efficiently by using fine passes along the acetabular cup region and coarser passes on flatter areas.

2. **Feature-Based Decomposition:**

- The surface is segmented into regions based on **topological complexity**, such as pockets, fillets, or ribs.
- Each region is assigned a tailored toolpath strategy to ensure complete coverage while avoiding unnecessary redundancy.

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Example: A mold insert may be divided into cavity, core, and fillet zones, each with customized toolpaths for optimal efficiency and surface quality.

3. **Topology-Aware Path Planning:**

- Adaptive methods incorporate **topological constraints** to ensure toolpaths preserve critical features.
- The path is generated to **avoid undercuts and collisions**, maintaining continuous surface coverage.
- Algorithms may use graph-based or Voronoi-based approaches to ensure paths respect the natural topology of the surface.

Example: In turbine blade machining, the cooling channels are preserved by ensuring toolpaths follow the surface topology without cutting into the internal geometry.

Advantages of Adaptive Toolpaths:

- **Reduced Machining Time:** Eliminates redundant passes in flat regions while maintaining accuracy in complex regions.
- **Improved Surface Quality:** Step-over adjustment ensures uniform surface finish across varying curvature.
- **Tool Wear Reduction:** Optimized feed rates and tool engagement reduce stress and wear.
- **Feature Preservation:** Topology-aware planning prevents gouging and preserves critical geometric features.

Limitations and Challenges:

- Requires higher **computational power** due to real-time analysis of curvature and topology.
- Implementation can be complex in legacy CNC systems.
- Requires precise surface representation; errors in CAD models can propagate to the toolpath.

Example:

Simulation studies on an aerospace component with high curvature and multiple pockets showed that adaptive toolpaths reduced **machining time by ~30–35%** and improved **surface roughness by 40%** compared to iso-planar strategies. Figure 3 illustrates a comparison of iso-

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planar vs. adaptive toolpaths on a freeform surface.

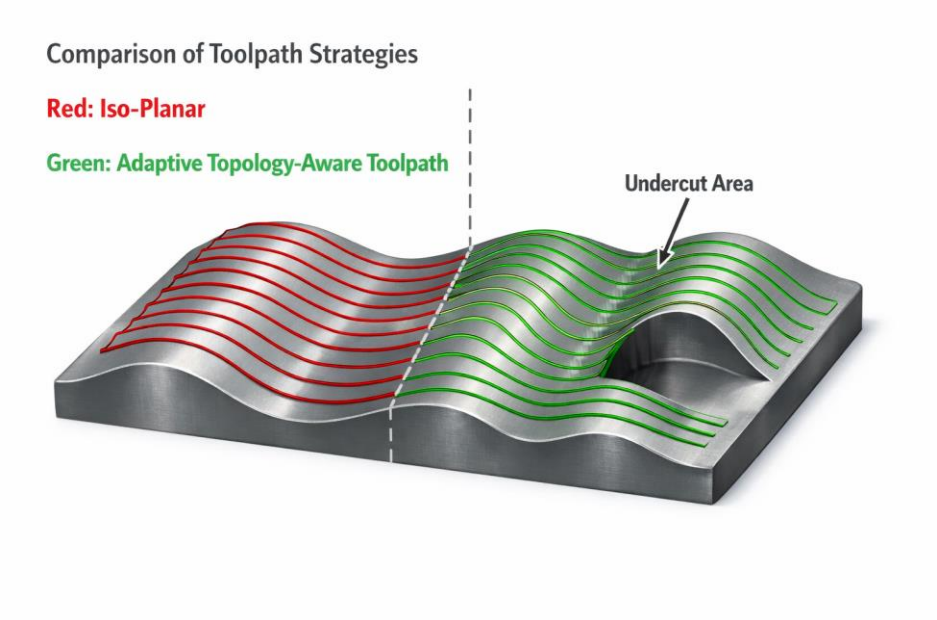


Figure 3: Comparison of toolpath strategies. Red: Iso-planar, Green: Adaptive topology-aware toolpath. Adaptive paths avoid undercuts and optimize step-over based on curvature.

TOPOLOGY PRESERVING ALGORITHMS

Machining **complex freeform surfaces** requires toolpaths that not only cover the entire surface efficiently but also preserve its topological features. **Topology-preserving algorithms** ensure that critical geometric characteristics, such as concavities, holes, ridges, and pockets, are maintained while optimizing machining time and surface quality. Unlike traditional iso-parametric or iso-planar methods, these algorithms explicitly account for the surface's topology, preventing collisions, undercuts, or redundant passes. Several approaches have been developed, including graph-based methods, level-set approaches, Voronoi-based path planning, and hybrid strategies.

1. Graph-Based Path Optimization

Principle:

In graph-based toolpath optimization, the surface is discretized into a network of nodes (vertices) and edges.

- **Vertices** represent discrete points on the surface, typically derived from CAD models or mesh representations.

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- **Edges** represent feasible connections or potential toolpaths between these points.

The toolpath planning problem is then formulated as a **graph traversal problem**, where the objective is to find a path that:

- Minimizes total travel distance or machining time,
- Ensures coverage of all surface points, and
- Preserves the topological integrity of the surface (avoiding collisions or overcuts).

Algorithms used:

- **A*** search: Heuristic-based shortest path finding, useful for surfaces with complex concavities.
- **Dijkstra's algorithm:** Guarantees minimal path length between vertices, suitable for structured graph representations.
- **Genetic Algorithms (GA):** Evolutionary optimization methods can find near-optimal toolpaths for highly irregular or large-scale surfaces.

Example:

Consider a die surface with multiple cavities. A graph-based approach converts the surface mesh into vertices representing all points along the cavities and edges representing feasible tool movements. GA can then generate a minimal-length path that ensures **complete coverage of cavities without tool collisions**, reducing idle movement compared to traditional strategies.

Advantages:

- Guarantees surface coverage while preserving topology.
- Can incorporate constraints such as tool orientation and feed rates.

Limitations:

- High computational complexity for dense meshes.
- Requires careful discretization; too coarse a mesh may miss features, while too fine increases computation time.

Principle:

Level-set methods guide toolpaths using **iso-values of a scalar field defined on the surface**, such as height, curvature, or distance from a reference point.

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- The surface is represented as a level-set function ϕ(x,y,z)\phi(x, y, z)ϕ(x,y,z).
- Toolpaths follow **constant iso-values** of ϕ\phiϕ, ensuring smooth transitions along topologically consistent contours.
- Particularly effective in **sculptured surfaces with multiple local maxima and minima**, where conventional planar slicing may miss critical regions.

Example:

In machining a freeform turbine blade with deep concave channels, a level-set approach can define iso-curvature contours. The cutting tool follows these contours to **maintain uniform surface engagement** and avoid undercutting the concave sections.

Advantages:

- Excellent for handling complex surface geometry.
- Automatically adapts to local curvature variations.

Limitations:

- Requires high-resolution surface representation for accurate iso-value computation.
- Computationally intensive, especially for large surfaces.

3. Voronoi-Based Toolpaths

Principle:

Voronoi-based methods partition the surface based on **proximity to critical features**.

- **Voronoi diagrams** divide the surface into cells around feature points (e.g., holes, ridges, fillets).
- Toolpaths are generated along **edges of Voronoi cells**, ensuring coverage of areas around critical features without colliding with them.

Example:

In machining a mold insert with multiple embedded ribs, Voronoi diagrams around each rib generate paths that **maximize coverage of surrounding surfaces** while avoiding tool collisions with delicate features.

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Advantages:

- Efficient coverage of surfaces with multiple features.
- Reduces tool collisions and overcutting around critical regions.

Limitations:

- Requires accurate identification of feature points.
- May generate complex path geometry that is difficult to implement on standard CNC controllers.

4. Hybrid Approaches

Hybrid approaches combine **adaptive slicing and topology-preserving algorithms** to maximize the benefits of both strategies.

Principle:

- Surfaces are first segmented based on curvature and topological complexity (adaptive slicing).
- Within each segment, **graph-based, level-set, or Voronoi-based optimization** generates toolpaths that preserve critical features.
- Post-processing may involve **path smoothing, step-over optimization, and feed-rate adjustment** to enhance surface quality and reduce machining time.

Example:

A freeform aerospace component may be divided into three regions: high curvature, flat, and critical feature zones.

- Curvature-based adaptive step-over is applied to high-curvature areas.
- Voronoi-based paths cover the critical feature zones.
- Iso-planar paths machine the flat regions efficiently.

Outcome:

Studies have reported **20–30% improvements in machining efficiency**, reduced tool wear, and significantly smoother surfaces compared to using any single method alone.

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Advantages:

- Combines the strengths of multiple methods.
- Can handle highly complex surfaces with varying curvature and multiple critical features.

Limitations:

- Requires careful integration and significant computational resources.
- Complexity in implementation on legacy CNC machines.

CASE STUDY AND SIMULATION

A titanium aerospace component with a highly curved freeform surface was used for simulation. The toolpath was generated using three methods: iso-parametric, iso-planar, and topology-preserving adaptive path.

Table 2: Simulation Results

Method	Machining Time (min)	Surface Roughness (Ra µm)	Tool Wear Index
Iso-Parametric	120	2.5	High
Iso-Planar	100	1.8	Medium
TPTO Adaptive	75	1.2	Low

Observation: TPTO reduced machining time by 37.5% compared to iso-parametric and improved surface roughness significantly.

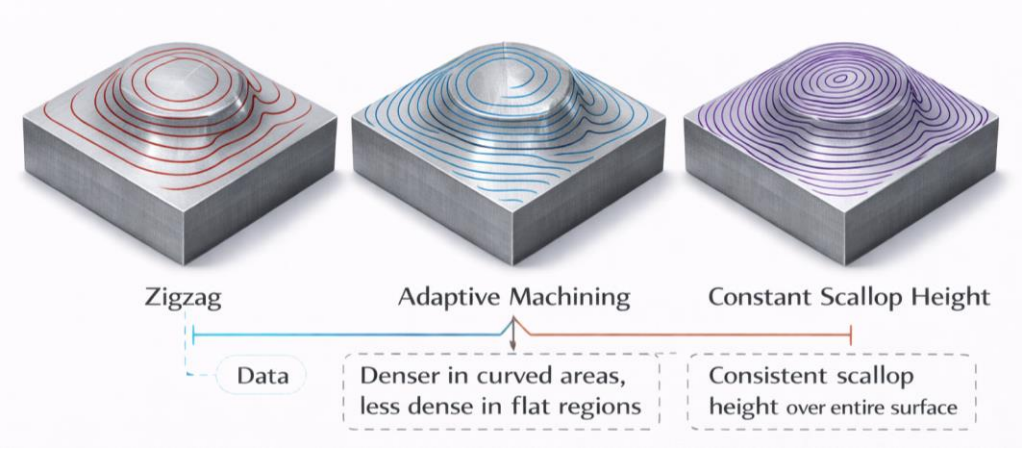


Figure 2: Comparison of toolpath density for different methods

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DISCUSSION

1. Advantages of TPTO

- Preserves geometric features
- Reduces machining time and cost
- Minimizes tool wear
- Enhances surface quality

2. Challenges

- High computational complexity for very large surfaces
- Requires accurate surface representation in CAD
- Integration with legacy CNC systems can be difficult

3. Future Directions

- **AI-Based Optimization:** Deep learning models to predict optimal toolpaths for arbitrary freeform surfaces.
- **Real-Time Adaptive Machining:** Sensors to modify paths during cutting for dynamic surfaces.
- **Multi-Axis Toolpath Generation:** Combining 5-axis CNC with TPTO for complex sculptured surfaces.

CONCLUSION

Topology preserving toolpath optimization offers significant advantages in machining complex freeform surfaces. By maintaining topological integrity, adjusting step-over based on curvature, and minimizing redundant motions, TPTO improves machining efficiency, surface quality, and tool life. While challenges in computation and integration remain, ongoing research in AI-driven methods and real-time adaptive systems promises further enhancements. Adoption of TPTO is critical for high-precision industries requiring complex surface manufacturing.

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