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MHD Effect on Solid fluid thermal Conductivity Ratio and Wall Thickness in a Nanofluid Filled Enclosure

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Abstract

The study of laminar magnetohydrodynamic (MHD) conjugate natural convection flow on an incompressible, viscous and electrically conducting fluid with heat conducting vertical wall and uniform heat flux is numerically investigated. In the analysis, a two-dimensional rectangular enclosure filled with water-Cu nanofluid is used under conjugate convective conduction condition. The left side wall of the enclosure is kept at a constant heat flux which is maintained at ambient air temperature while a constant low temperature is used on the right wall. The horizontal walls are adiabatic. The governing model has been solved by using finite element method with Galerkin weighted residual simulation. The objective of the study is to examine the momentum and energy transport processes in a rectangular enclosure in presence of magnetic field. The outcome is shown in terms of parametric presentations of streamlines and isotherms for the parameter solid fluid thermal conductivity ratio (K_r) *and solid wall thickness* (w_1) *on heat transfer and fluid flow inside the cavity for the range of Hartmann number (Ha) of 0 to 60. Moreover, the implications of the above parameters are depicted on the average Nusselt number (Nu) of the fluid. Finally, it is found that heat transfer and fluid flow can be controlled by the thickness of the solid wall and the thermal conductivity ratio.*

Keywords: *Magnetohydrodynamics, Conjugate heat transfer, Nanofluid, Finite element, Enclosure.*

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Nomenclature

C_p Specific heat capacity (J/K)

B_0 magnetic field stregth (Wb/m^2)

g gravitational acceleration (m/s^2)

Gr Grash of number $(\beta g H^4 q''/k\upsilon^2)$

H height of cavity (m)

h_∞ convective heat transfer coefficient (W/ m^{2}K)

Ha Hartmann number

L width of cavity (m)

l_1 distance between the left wall and the divider (m)

K thermal conductivity (W/mK)

K_r conductivity ratio $\left(K_r = \frac{\kappa_w}{k_f}\right)$

Nu Nusselt number

Nu_l local Nusselt number

Nu_{av} average Nusselt number

Pr Prandtl number, $\frac{\nu_f}{\alpha_f}$

q''or q heat generation per area,(W/ m^2)

Ra Rayleigh number ($\beta g H^4 q''/k\nu\alpha$)

T temperature (K)

T_∞ ambient air flow

X,Y dimensionless Cartesian coordinates,

$X = \frac{x}{L}$, $Y = \frac{y}{L}$

u,v components of velocity (m/s)

U, V dimensionless velocity components

w width of the thick wall (m)

w_1 dimensionless width of the thick wall

Greek symbols

α thermal diffusivity (m^{2}s^{-1})

β coefficient of thermal expansion (K^{-1})

ϕ solid volume fraction

ΔT Ref. temperature difference

Θ dimensionless fluid temperature

μ dynamic viscosity of the fluid (m^{2}s^{-1})

ν kinematic viscosity of the fluid (m^{2}s^{-1})

ρ density of the fluid (kgm^{-3})

Σ fluid electrical conductivity (Ω^{-1}.m^{-1})

ψ stream function

Subscripts

av	average
c	cold wall
f	pure fluid
nf	nanofluid
p	nanoparticles
s	solid
w	wall

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I. INTRODUCTION

Conjugate heat transfer occurs when the temperature field in the fluid is coupled to the temperature field in the solid. Heat transfer in solids and fluids are combined in the majority of applications to progress thermal performances of a given process. A perfect explanation of geometrical configurations, heat transfer modes, flow regimes and objects properties enables the investigation of temperature fields and heat transfer accurately. It is the preliminary point to test different configurations for a numerical model that can be used to predict conjugate heat transfer effects.

The natural convection under the influence of a magnetic field has many engineering applications. Sparrow and Cess [1] studied effect of magnetic field on a laminar boundary layer flow with an isothermal vertical plate. Magnetohydrodynamic natural convection in electrically conducting fluid on a heated vertical flat plate was investigated by Riley [2] and also developed an outer and inner solution in the moving layer of the fluid. Kuiken [3].numerically analyzed free convection in strong flow field under the influence of magnetic field for both the liquid metals and ionized gases. Oreper and Szekely [4] considered the effect of an externally applied magnetic field in a rectangular cavity on buoyancy motivated flow and observed the transient growth of fluid flow and temperature field in the enclosure. They establish that the presence of a magnetic field could repress natural convection flow as well as the power of the magnetic field. Alchaar et al. [5] studied a transverse magnetic field in a rectangular enclosure on natural convection flow. . Rudraiah et al. [6] investigated the effect of surface tension on buoyancy driven flow of an electrically conducting fluid in a rectangular cavity in the presence of a vertical transverse magnetic field. They saw how force damped hydrodynamics movement was found. Rudruiah et al. [7] also studied the effect of a magnetic field on free convection in a rectangular enclosure. All these analysis demonstrate that there has been a growing interest to comprehend the flow behavior and the heat transfer mechanism filled with electrically conducting fluids enclosure and are in the influence of a magnetic field.

Moreover, convection heat transfer can be controlled by the magnetic field was found by Dulikravich and Colaca [8]. The effect of magnetic field and heat generation along a vertical flat plate on natural convection flow in presence of conduction

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was studied by Mamun et al. [9]. They found that the temperature of the fluid increase with the increase of magnetic field and heat generation parameter. They also concluded that heat transfer decrease with the increase of magnetic field. However, the effect of nanoparticles on heat transfer enhancement in natural convection was conducted by Aminossadati and Ghasemi [10]. In this study based on cooling performance of the fluid the influence of Rayleigh number, location of heat source and solid volume fraction of nanoparticles are analysied numerically. A numerical simulation of conduction on combined convection heat transfer divided by perpendicular solid partition in a lid driven enclosure has been investigated by Oztop et al. [11].

The double-diffusion convective flow in a rectangular enclosure was studied by Teamah [12]. This investigation concluded that the heat and mass transfer mechanism and the flow characteristics inside the enclosure strongly depend on the strength of the magnetic field and the heat generation. Bilir and Ates [13] investigated transient conjugate heat transfer on an axial fluid flow in a thick walled duct with convective boundary conditions. Sivasankaran and Ho [14] numerically studied the significance of temperature dependent properties in a water filled cavity on the natural convection flow under the influence of a magnetic field. They conclude that the heat transfer rate is influenced by the direction of the external magnetic field and decrease with an increase of the magnetic field. Conduction–convection heat transfer on a steady state condition was investigated by Mobedi and Sunden [15]. They used a vertical plate fin with small heat source. They also found that for small conduction convection parameter, fin act as isothermal vertical plate and the heat transfer rate does not vary with the location of heat source. But the heat transfer rate is influenced for large value of conduction convection parameter and the location of heat source.

Rahman and Alim [16] investigated MHD combined convection flow containing a heat conducting circular cylinder in a vertical lid-driven square enclosure in presence of Joule heating. Their result indicated that Hartmann number and Reynolds number have strong control on the flow and thermal behavior of the fluid and the characteristic of heat transfer mechanism strongly depend upon the strength of the fluid. An experiment investigation was done by Wu and Ching [17] on natural convection in a square

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cavity filled with air with top wall partition. They found that nusselt number and ambient temperature near the corner area affected by the partition height due to rectangular flow and thermal effect on the surface Effect of Magnetic field on natural convection in a water based nanofluid filled square enclosure bounded by two isothermal vertical walls was .studied by Ghasemi et al. [18]. The result of this investigation showed that with the increase of Rayleigh number heat transfer rate increase but it reduce with an increase of Hartmann number. It is also seen from the study that adding of nanoparticals results in enhancement of fluid thermal behavior.. Effect of conduction and convection on MHD boundary layer flow with heat generation was studied by Salina and Alim [19].

Karatas and Derbentli [20] experimentally investigated natural convection in rectangular cavity. They carried out an experimental study in a rectangular prism shape cavity on considering one active vertical wall. Nemati et al. [21] used Lattice Boltzmann model to show the effect of magnetic field on natural convection flow in a rectangular cavity filled with nanofluid.. Effect of joule heating on natural convection in non-linearly heated square enclosure was investigated by Oztop and Salem [22]. They found that flow becomes weaker near the right corner of the cavity due to non-isothermal boundary condition. They also showed that both Hartmann number and joule heating parameter have significant effects on heat transfer and fluid flow.

Patil et al. [23] deliberated conjugate effect and surface radiation on laminar convection flow in an Protrusion shaped enclosure. They observed that the protrusion affects significantly the overall heat transfer process and modify the flow and temperature field. Solid fluid interaction in a square cavity with a heat conductive portion on natural convection was studied by Jamesahar et al. [24]. They considered a partitioned cavity divided to two sub cavity by a flexible membrane. They found that the finite thermal conductivity of the membrane affect the heat transfer in the cavity.

Seremet et al. [25] investigated an inclined wavy enclosure with corner heater in MHD natural convection flow. They focus on the effect of Hartmann number on the flow pattern and heat transfer inside the enclosure. In another study

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Khatamifar et al. [26] made a numerical study in a differentially heated square cavity having finite partition and thermal conductivity on conjugate natural convection flow. They established that average nusselt number raise with Rayleigh number and shrink with partition thickness.

Sivaroj and Sheremet [27] investigate natural convection in an inclined porous cavity containing a heat conducting solid located at the center under the influence of imposed magnetic field. .They investigate in detail the effect of inclination angle of the magnetic field and the Hartmann number on the flow and the thermal field; they also concluded magnetic field reduces the convective heat transfer rate in the cavity, they also found that with the increase in the angle of the magnetic field results a non-linear difference in the average Nusselt numbers.

Various non-square shapes on natural convection in fluid media, porous media and nanofluid filled enclosure were studied by Das et al. [28]. They present helpful tools for the researchers in choosing the realistic enclosures and suitable process for the convective system.

In recent time, John Hyeon Son and Seouk Park [29] examined numerically laminar natural convection flow in a differently heated rectangular enclosure with an insulated square block in the presence of horizontally applied magnetic field. They also found that heat transfer decrease with the increase of Hartmann number, they also conclude that insulated block contributes to enhance heat transfer rate for a certain range of the block size.

Zargartalebi et al. [30] analyzed unsteady conjugate natural convection in a porous cavity with finite heat conductive walls. They considered time periodic boundary condition and thermal non-equilibrium. Their result illustrated that for non dimensional frequency, wall thickness and the amplitude of periodic fluid, nusselt number is an increasing function of Rayleigh number and thermal conductivity. They also investigated nusselt number under non thermal equilibrium condition.

As discussed before, the magnetic field reduces the convective circulating flows within the enclosure filled with electrically conducting fluid, this results the reduction of heat transfer. The addition of nanoparticles to the fluid can improve its thermal performance and progress the heat

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transfer mechanism in the enclosure. Hence the present investigation numerically examined the effect of Hartmann number on thermal conductivity ratio and wall thickness under the influence of horizontally applied magnetic field.

2. GEOMETRICAL MODELING

A two dimensional rectangular enclosure filled with electrically conducting fluid with sides of width L and height H is depicted in the Figure 1. The enclosure is filled with a water-based nanofluid including copper nanoparticles .The nanofluid is considered as Newtonian, incompressible and the flow is regarded as laminar. In the enclosure, a constant heat flux q is applied on left side which is maintained at ambient air temperature. On the other hand, a low temperature T_c is used on the right side. The top and bottom walls are adiabatic.

The left wall has a thickness of width $w_1 = \frac{w}{\cdot} = 0.1$, , while the thicknesses of the other boundaries of the wall are assumed to be zero. A movable conducting divider is attached to the horizontal bottom wall of the cavity. A magnetic field of strength B_0 is applied horizontally normal to the side walls. The base fluid and nanoparticles (Cu) are in thermal equilibrium and no slip maintains among them.

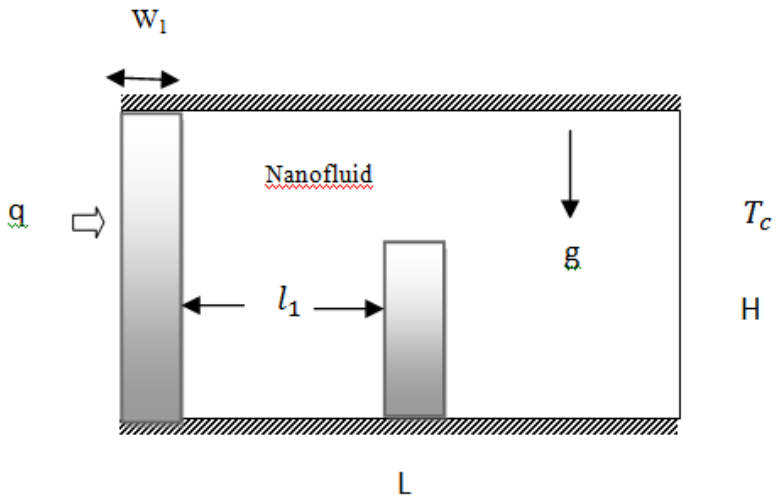


Figure.1: Geometry of the model

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3. MATHEMATICAL EXPRESSION

The leading equations governing the viscous incompressible flow and the temperature distribution with the Boussinesq approximations inside the cavity are described by the Navier–Stokes and the energy equations [10] can be the present as follows:

For Fluid:

Continuity Equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum Equations:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_{nf}}\left[-\frac{\partial p}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right] \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho_{nf}}\left[-\frac{\partial p}{\partial y}+\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+(\rho\beta)_{nf}g(T-T_c)-\sigma_{nf}B_o^2 v\right] \tag{3}$$

Energy conservation Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

For Solid:

Energy conservation Equation:

$$\frac{\partial^2 T_w}{\partial x^2}+\frac{\partial^2 T_w}{\partial y^2}=0 \tag{5}$$

where the effective density of the nanofluid is given as,

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_p$$

adding up the thermal diffusivity of the nanofluid is,

$$\alpha_{nf}=\frac{\kappa_{nf}}{(\rho\, C_p)_{nf}}$$

the heat capacity of the nanofluid and expressed as,

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_p$$

The thermal expansion coefficient of the nanofluid is expressed as,

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_p$$

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In the current study, the viscosity of the nanofluid is considered by Brinkmann [31] is given as,

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$$

The thermal conductivity which for spherical nanoparticles, according to Maxwell model [32] is,

$$\kappa_{nf} = \kappa_f \left[\frac{(\kappa_p + 2\kappa_f) - 2\phi(\kappa_f - \kappa_p)}{(\kappa_p + 2\kappa_f) + \phi(\kappa_f - \kappa_p)}\right]$$

where κ_n is the thermal conductivity of dispersed nanoparticles and κ_f is the thermal conductivity of pure fluid.

The boundary conditions for the present problem are specified as follows:

For all rigid walls: $u = v = 0$

At the divider surface: $u = v = 0$

At the left side of thick wall:

$$q = -\kappa_w \frac{\partial T}{\partial x}|_w + h_\infty (T_w - T_\infty)$$

At the right vertical wall: $T = T_c$

At the top and bottom walls: $\frac{\partial T}{\partial y} = 0$

At the fluid solid wall interfaces: $\kappa_w \frac{\partial T}{\partial x}|_w = \kappa_{nf} \frac{\partial T}{\partial x}|_{nf}$

So as to estimate heat transfer enhancement, we have calculated the local Nusselt number and average Nusselt number at the thick wall (hot wall) as

$$Nu_l = -\frac{\kappa_{nf}}{\kappa_f} \frac{H}{(T_w - T_c)} \frac{\partial T}{\partial x}|_{x=0.1}$$

The average Nusselt number is given by $Nu_{av} = \int_0^H Nu_l dy$.

The above equations are non-dimensionalized upon incorporating the following dimensionless variables:

$X = \frac{x}{L}$, $Y = \frac{y}{L}$, $U = \frac{uL}{\alpha_f}$, $V = \frac{vL}{\alpha_f}$, $P = \frac{\bar{p}L^2}{\rho_{nf}\alpha_f^2}$, $\theta = \frac{T - T_c}{\Delta T}$ and $\Delta T = \frac{q''L}{\kappa_f}$.

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Once apply these non-dimensional variables the system of equations governing the flow using conservation of mass, momentum and energy equations for nanofluid and solid in non-dimensional form can be written as follows:

For Fluid:

Continuity Equation:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{6}$$

Momentum Equations:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{7}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}\,Ra\,\mathrm{Pr}\,\theta-Ha^2\,\mathrm{Pr}\,V \tag{8}$$

Energy Equation:

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{9}$$

For Solid:

$$\frac{\partial^2 \theta_w}{\partial X^2}+\frac{\partial^2 \theta_w}{\partial Y^2}=0 \tag{10}$$

The dimensionless parameter appearing in the eqs (7)-(10) are as follows:

Rayleigh numbe $=\frac{g\,\beta_f L^3 \Delta T}{\nu_f \alpha_f}$, Hartmann number $Ha=B_o L\sqrt{\frac{\sigma_{nf}}{\rho_{nf}\nu_f}}$, B_o is the magnitude of the magnetic field and σ is the electrical conductivity and Prandlt number $Pr=\frac{\nu_f}{\alpha_f}$,where $\alpha=\frac{\kappa}{\rho C_p}$, is the thermal diffusivity of the fluid.

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The non dimensional boundary conditions under consideration can be written as:

For all rigid walls: $U = V = 0$

At the divider surface: $U = V = 0$

At the left side of thick wall: $q = -\frac{\kappa_w}{\kappa_{nf}}\frac{\partial\theta}{\partial X}|_w + \frac{H}{\kappa_{nf}}\square_\infty\theta_w = 1$

At the right vertical wall: $\theta = 0$

At the top and bottom walls: $\frac{\partial\theta}{\partial Y} = 0$

At the fluid solid wall interfaces: $\frac{\partial\theta_{nf}}{\partial X} = K_r\frac{\partial\theta_w}{\partial X}$

Where, $K_r = \frac{\kappa_w}{\kappa_{nf}}$ is the solid fluid thermal conductivity ratio.

The local Nusselt number at the thick wall on the enclosure base on the non dimensional variable can be expressed as,

$$Nu_l = -\frac{\kappa_{nf}}{\kappa_f}\frac{\partial\theta}{\partial X}|_{x=0.1}$$

and the average Nusselt number at the right thick wall as

$$Nu_{av} = -\frac{\kappa_{nf}}{\kappa_f}\int_0^H \frac{\partial\theta}{\partial X}dy\,.$$

4. NUMERICAL IMPLEMENTATION

To solve the non-dimensional governing equations (2) - (5) along with boundary conditions the Galerkin finite element method [33, 34] are used for the considered problem. The continuity equation is automatically fulfilled for large values of this constraint. Then the velocity components (U, V) and temperature (θ) are expanded using a basis set. Three points Gaussian quadrature is used to evaluate the integrals in these equations. Then the non-linear residual equations are solved using Newton-Raphson method to determine the coefficients of the expansions.

The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion such that $|\psi^{n+1} - \psi^n| \leq 10^{-4}$, where n is the number of iteration and ψ is a function of U, V and θ.

4.1 Grid Generation

Grid or mesh generation is the partition of the geometry model into small units of

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simple shapes named finite elements, control volume etc that approximates the physical domain in finite element method. Dependent variables are approximated at the local element coordinates defined by the numerical grid. It is mainly a disconnected demonstration of the physical domain where the solutions are to be carried out. Meshing the complicated geometry make the finite element method a powerful technique to solve boundary value problems occurring in a range of engineering applications. Figure 2 shows the mesh configuration of present physical domain with triangular finite elements.

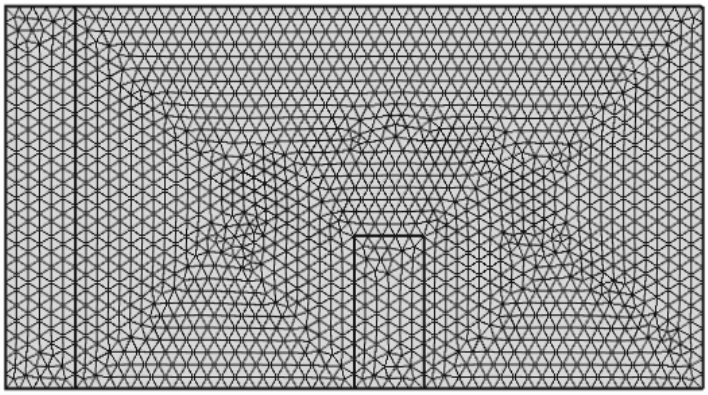


Figure 2 Mesh Structure of element for the model

4.2 Thermo-Physical Properties

The thermo-physical properties of the nanofluid are taken from Aminossadati and Ghasemi [10] given in Table 1.

TABLE 1: Thermo physical properties in this study

Physical Properties	Water	Cu
C_p(J/kgK)	4179	385
ρ (kg/m^3)	997.1	8933
k (W/mK)	0.613	401
$\beta\times10^5$ (K^{-1})	21	1.67

4.3. Grid Independent Test

A grid independence test is conducted with five types of mesh for $Ra = 10^6,\ Ha = 30,\ \phi = 0.05, l_1 = 0.40, Pr = 6.2,\ \ h_{nf} = 100\ W/m^2K\ \ ,K_r = 10$ and

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$w_1 = 0.1$ through an rectangular enclosure. **Table 2** shows the Nusselt quantities on the grid size. The numerical scheme is carried for water-copper nanofluid (ϕ = 5%) as well as base fluid (ϕ = 0%) for the aforesaid elements to develop an understanding of the grid fineness as shown Figure. 3. The scale of the average Nusselt number for 3258 elements shows a little difference with the results obtained for the other elements. Hence, considering the non-uniform grid system of 3258 elements is preferred for the computation.

Table 2: Grid Test at $Ra = 10^6, \phi = 0.05, l_1 = 0.40, Pr = 6.2, , h_\infty = 100 \frac{W}{m^2K}$ $, K_r = 10$ $Ha = 30$ ***and*** $w_1 = 0.1$

Nodes (elements)	6181 (394)	10062 (533)	13922 (1058)	16181 (3258)	28926 (13020)
Nu (nanofluid)	2.81063	2.81877	2.92582	3.01603	3.01899
Nu (basefluid)	2.71233	2.72012	2.84810	2.95562	2.95577
Time [s]	96.265	106.594	192.157	256.328	390.377

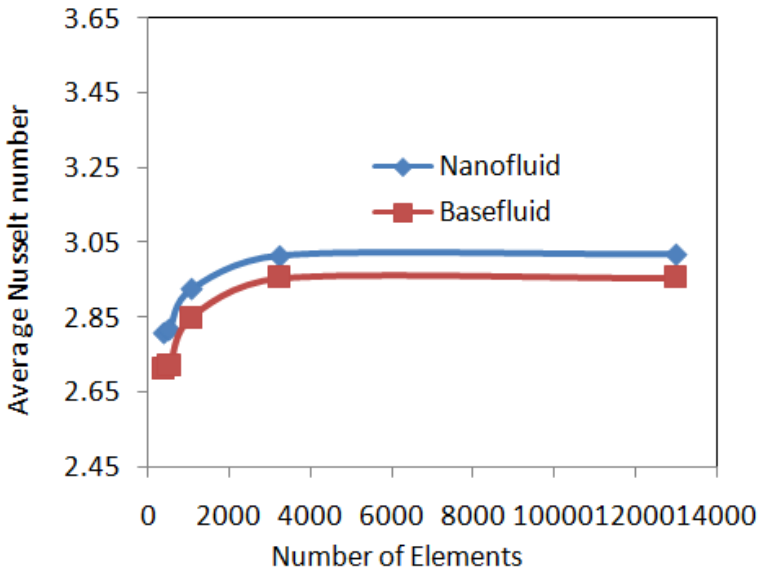


Figure 3: Grid Refinement check

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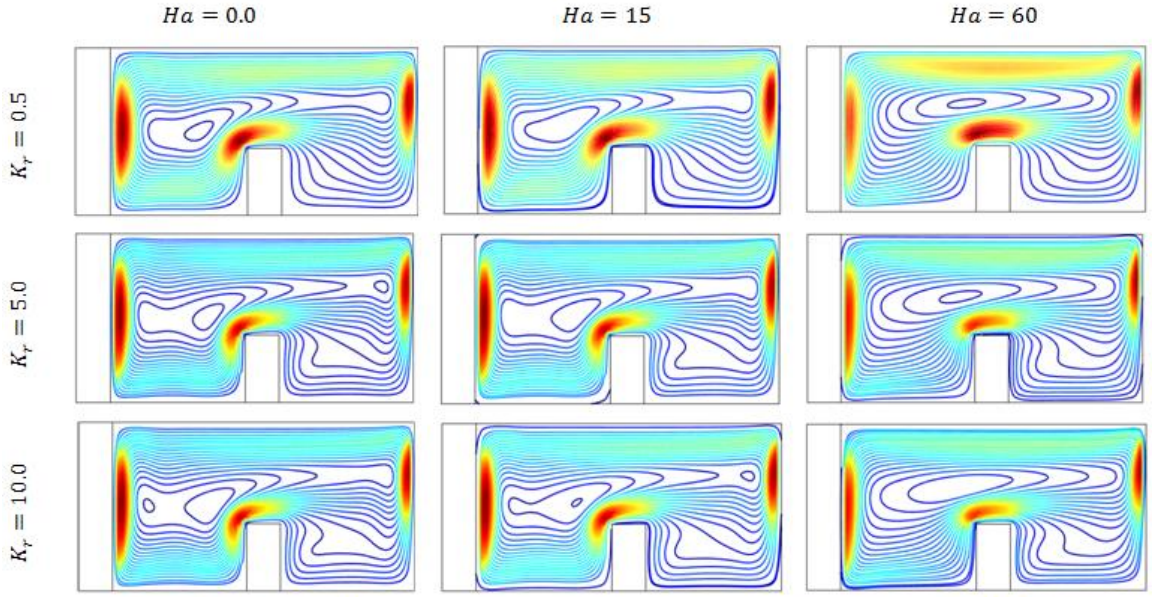


Figure 4: (a) Effect of Hartmann number $(Ha = 0.0, 15, 60)$ on Streamlines at various Solid Fluid thermal conductivity ratio for $Ra = 10^6, Pr = 6.2,\ h_\infty = 100W/m^2\mathrm{K}$, and $w_1 = 0.1$.

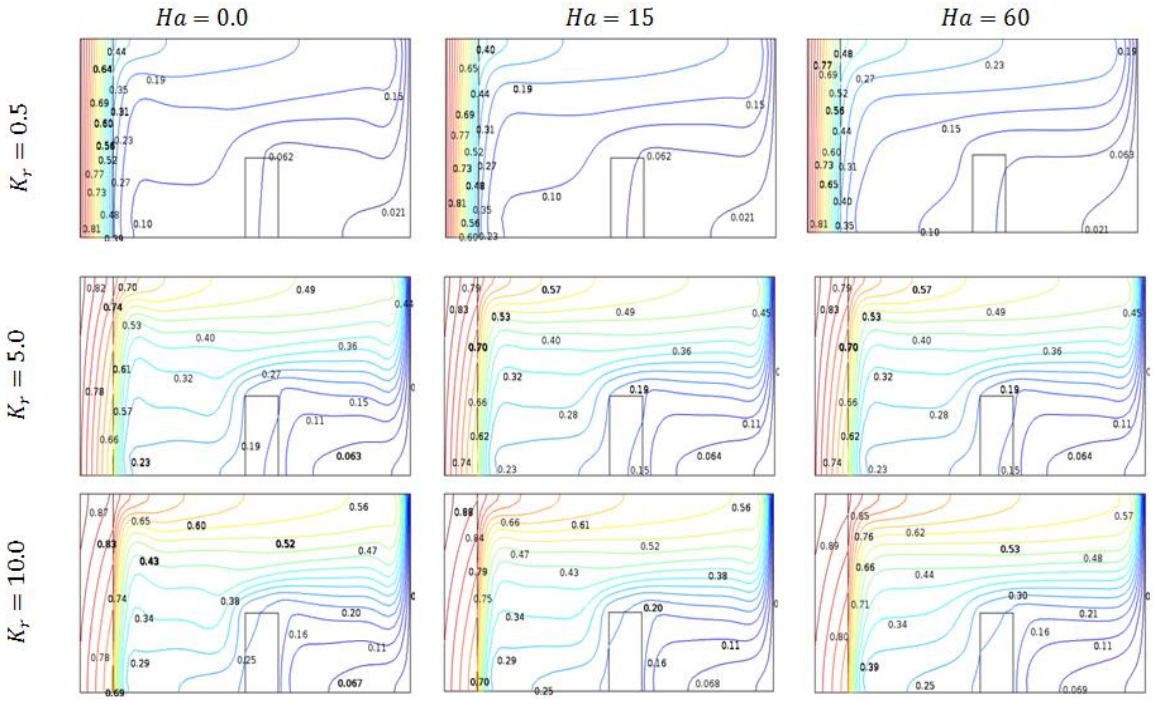


Figure 4: (b) Effect of Hartmann number $(Ha = 0.0, 15, 60)$ ***on Isothermss at various Solid Fluid thermal conductivity ratio for*** $Ra = 10^6, Pr = 6.2,\ h_\infty = 100W/m^2\mathrm{K}$, ***and*** $w_1 = 0.1$.

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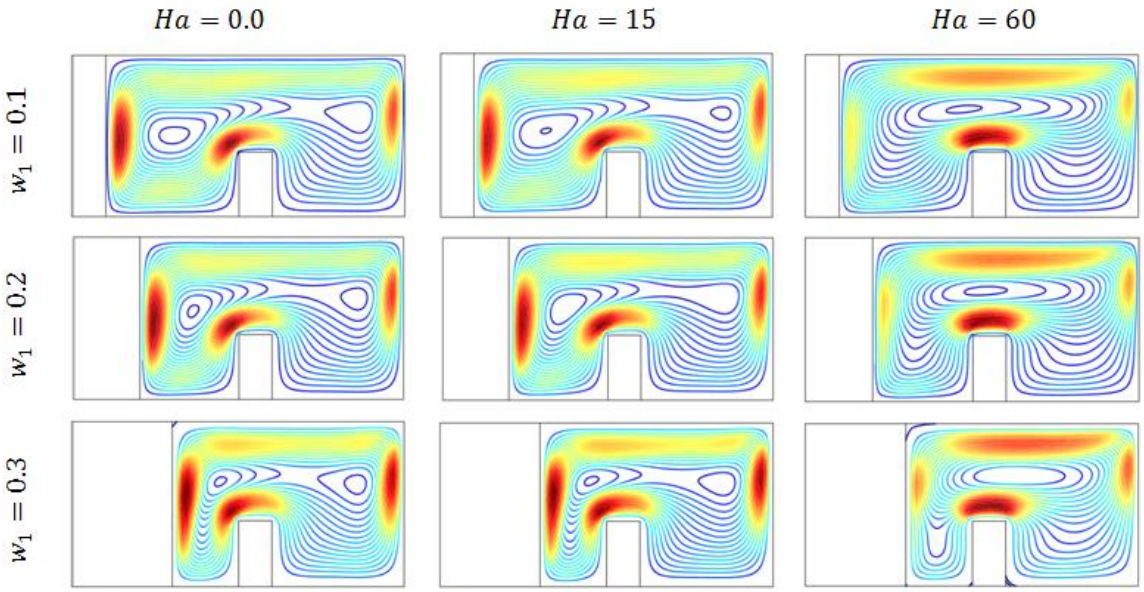


Figure 5: (a) Effect of Hartmann number $(Ha = 0.0, 15, 60)$ on Streamlines at various Solid wall thickness for $Ra = 10^6, Pr = 6.2,\ h_\infty = 100 W/m^2\text{K}$, $K_r = 10$ and $l_1 = 0.4$.

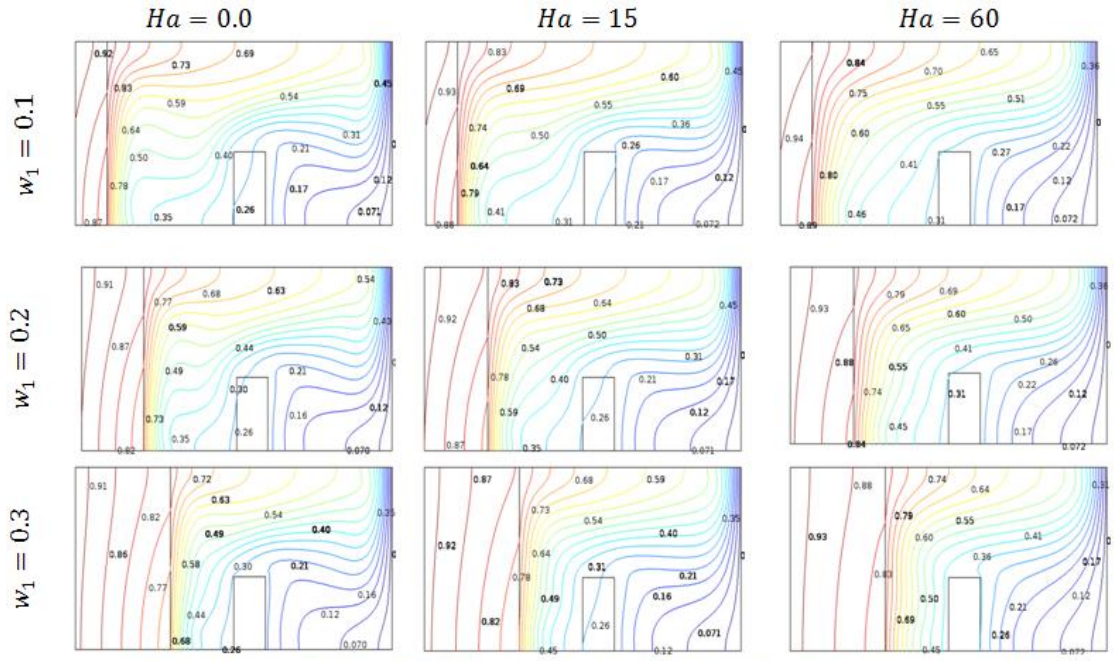


Figure 5: (b) Effect of Hartmann number $(Ha = 0.0, 15, 60)$ on Isotherms at various Solid wall thickness for $Ra = 10^6, Pr = 6.2,\ h_\infty = 100 W/m^2\text{K}$, $K_r = 10$ and $l_1 = 0.4$.

5. RESULTS AND DISCUSSION

In this section, the effects of MHD on conjugate natural convection heat transfer in a rectangular enclosure with heat conducting vertical wall and uniform heat flux are explored with solid volume fraction $\phi = 5\%$. Numerical results in terms of streamlines, isotherms for solid fluid thermal conductivity ratio (K_r) and solid wall thickness (w_1) on heat transfer

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and fluid flow inside the cavity have been studied for the range of Hartmann number (Ha) of 0 to 60.

The ranges of K_r and w_1 for this investigation vary from 0.5 to 10.0, 0.1 to 0.3 respectively. In addition, the values of the average Nusselt number (Nu) in the domain have been calculated for the above mentioned parameters. The outcomes for the different cases are presented in the following sections.

The influences of Hartmann number on the streamlines and isotherms for different values of solid fluid thermal conductivity ratio (K_r) with $Ra = 10^6, w_1 = 0.1, l_1 = 0.40, Pr = 6.2$ and $h_\infty = 100\text{W}/m^2\text{K}$ with $\phi = 0.05$ are depicted in Figures 4(a) and (b). In every case we noticed, the flow rises along the left straight up wall, then it get blocked to the top adiabatic wall, which then turn towards the right cold wall and get blocked again by the bottom wall, then it finally directed the flow towards the left wall. So a clockwise movement is located within the rectangular enclosure.

It is also seen that at the left side of the divider the centre of recirculation cell is found. We know at very low thermal wall conductivity ratio $(K_r = 0.5)$, the solid wall acts as a thermal resistance wall, hence a steeper temperature gradient of isotherm is observed here in the solid wall, which indicates less amount of heat transfer to the enclosure.

With the increase of wall thermal conductivity ratio the flow strength becomes higher and buoyancy dominates the flow. It is also noticeable in Figure 4 (b) that the isotherm becomes horizontal with the increase of thermal conductivity of the solid wall. The effect of Hartmann number (Ha) is more visible here. As we observe with the increase of Hartmann number (Ha) the magnetic field retards the buoyancy flow, and reduces the flow strength. So we see weaker flow pattern on the flow and the temperature field.

Figures 5 (a) and (b) examines the effect of Hartmann number (Ha) at various values of Solid wall thickness $(w_1 = 0.1, 0.2, 0.3)$ on the fluid flow and temperature field for $Ra = 10^6, l_1 = 0.40,\ Pr = 6.2$ and $h_\infty = 100\text{W}/m^2\text{K}$ with $\phi = 0.05$. We see that the parameter wall thickness (w_1) affects the fluid and solid wall temperature as well as the flow characteristics. Here we notice that the strength of the circulating cell is much higher for a thin solid wall. When the

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thickness of the solid wall increases it acts as a resistance wall, so less heat is transferred from the solid wall to the enclosure for higher value of solid wall thickness(w_1).

We also observe that two core are created in the enclosure for lower wall thickness and it becomes turn as elliptic shape as solid wall thickness increase, which is an indication of weaker convection flow. As in the case of isotherms we see with the increase of solid wall thickness from $w_1 = 0.1$ to $w_1 = 0.3$ the temperature gradient within the wall decreases, which make the less convection amount within the enclosure.

When the Hartmann number (Ha) applies to the fluid, the strength of the flow circulations decrease with increase of the Hartmann number (Ha) . As discussed earlier the magnetic field results in the decrease of convective circulating flows within the enclosures filled with electrically conducting fluid, this turn the reduction of heat transfer, which is also noticeable in the isotherms curves.

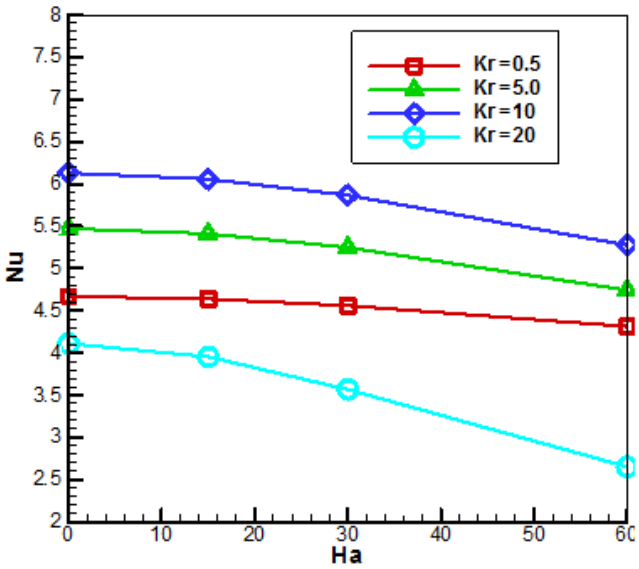


Figure 6 (a): Average Nusselt number at various K_r

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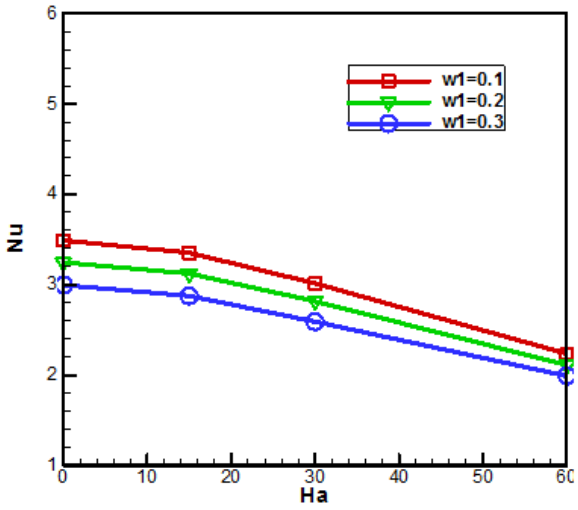


Figure 6 (b): Average Nusselt number at various w_1

The variation of average Nusselt number (Nu) with the Hartmann number (Ha) is displayed on Figure 6 (a) for the different values of solid fluid thermal conductivity ratio (K_r).We see that heat transfer increases with the increase of wall thermal conductivity ratio(K_r), where the increase of Hartmann number (Ha) results in the reduction of heat transfer performance of the enclosure. This is due to the greater effect of the magnetic field and the stronger suppression of the buoyancy flow in the nanofluid.

An examination of average Nusselt number (Nu) with respect to the Hartmann number $(Ha)$are seen for three values of Solid wall thickness (w_1) on taking $Ra = 10^6, l_1 = 0.40,\ Pr = 6.2$ and $h_\infty = 100\text{W}/m^2\text{K}$ with $\phi = 0.05$ on Figure 6 (b). We observe that the average Nusselt number (Nu) decreases with the increase of Solid wall thickness (w_1) and Hartmann number (Ha)due to the effect of wall thermal resistance and magnetic field.

CONCLUSION

Considering the results and discussion the following conclusions are made.

- Because of the temperature difference between the interface and the cold wall the fluid inside the cavity is driven. This difference is lower for walls, with poor thermal conductivity, but it becomes more significant with the raise of Kr, and leads to increase the average Nusselt number. Here it is found that heat transfer decreases with an increase of the magnetic field. So,

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convective heat transfer can be prohibited by the magnetic field.

- The natural convection inside the nanofluid filled cavity reduce with mounting of its wall thickness and by the influence of applied magnetic field due to thermal resistance of solid wall and the suppression of convective circulating flow. As a result average Nusselt number also reduced.

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