MANTECH Publications

Journal of Engineering Mathematics & Statistics
Volume 6, Issue 2, July-December 2022

An Example of the Real-World Application of Modeling Reliability in Industry

Dr. B. R Narayan[1], Vishal Raghuvansi[2]

Professor[1], Student[2]

Department of Mathematics

Patel College of Science & Technology

***Corresponding Author's Email**: raghuvanshivishal4@gmail.com[2]*

Abstract

The purpose of this essay is to apply statistical approaches to dependability difficulties using an experiment from the aerospace sector. To that purpose, failure data from this region is collected and categorised into two separate groups, with the category of interest being failures happening on components that will need to be changed in order to resolve the technical issues. In order to represent lifespan data from the category of interest, a Weibull distribution is presented, which is based on a non-parametric technique (the so-called Median Rank) for estimating cumulative distribution functions (c. d. f.), and some practical findings are provided using two basic instances.

Keywords: - *line replaceable unit, Weibull distribution, Median Rank, time to first failure*

INTRODUCTION

Statistics play an important role in reliability engineering by modelling lifespan distributions. The purpose of this study is to use inferential statistical approaches to data collected from field failures of turbofan aircraft engines, a kind of air breathing jet engine that is widely employed in aviation propulsion. Exhaustive lists of removals (i.e. lists that include all removals) are utilised for this purpose. Other events that are not deletions are not considered failures in this study since they might represent system modifications or software rollovers, and so do not involve defective systems. In general, faults can be divided into two broad

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categories: internal to the system (referring to cases where there is a system problem involving a line replaceable unit (LRU), and no fault found (NFF), which includes faults that are not due to the system itself and thus considered external to the system. We now give a short description of the aforementioned categories for clarity. A basic definition relating the LRU is that " it is a critical support item which is withdrawn and replaced at the field level in order to return the end item to an operational ready condition". This is defined as "a modular component of an aeroplane designed to be rapidly replaced at an operational site" in the aerospace industry. A broad definition of the NFF events is as follows. "An NFF arises when a reported failure cannot be recreated in the avionics shop and, as a result, cannot be repaired. The faulty box is returned to service but later reappears for testing. Ultimately, if the problem cannot be discovered and repaired after repeated tries the unit is branded as a rogue actor". This category also includes events caused by random variation, such as faulty electrical connections.

In the following sections, we explore such failures in a specific experiment and give some commentary on our findings using appropriate statistical approaches.

DATA CLASSIFICATION

Our experiment consists on documenting failure times using the time to first failure (TTFF) as a measure of reliability, which are really the time to first removal, where the removal is due to either an issue with the LRU or it is categorised as an NFF. Take note that these times are reported in unit hours and correspond to operation hours (so hours on the ground are not counted in even if the system is powered up). The information was gathered when the author was a research fellow at Strathclyde University (U.K.). First, it is worth investigating if this data is derived from a single population or whether there is a clear separation between the LRU and NFF categories. Figure 1 shows boxplots for the LRU and NFF lifespan groups, with sample sizes of order 21 for LRU and 28 for NFF, and they demonstrate distinct forms for the distributions of these groups, with the LRU distribution having a significantly more asymmetric shape. Also, the distributions' locations appear to be different, with LRU lives being generally shorter than those of NFF (however remark that there are two very high values of LRU lifetimes and the graph shows that these values are outliers). The Mann- Whitney U test is then employed, and because the sample sizes are more than 20, a normal approximation for U is used (see [3]), providing a result of $Z=2.4$ under the null hypothesis of

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lifespan equality in the two groups. We discover that |Z| is greater than 1.96, the crucial value from N(0,1) tables at the 5% significance level, and therefore the null hypothesis is rejected in favour of the alternative, meaning that the group distributions are significantly different. Because the distributions of LRU and NFF failure periods differ, using them as a single population is incorrect.

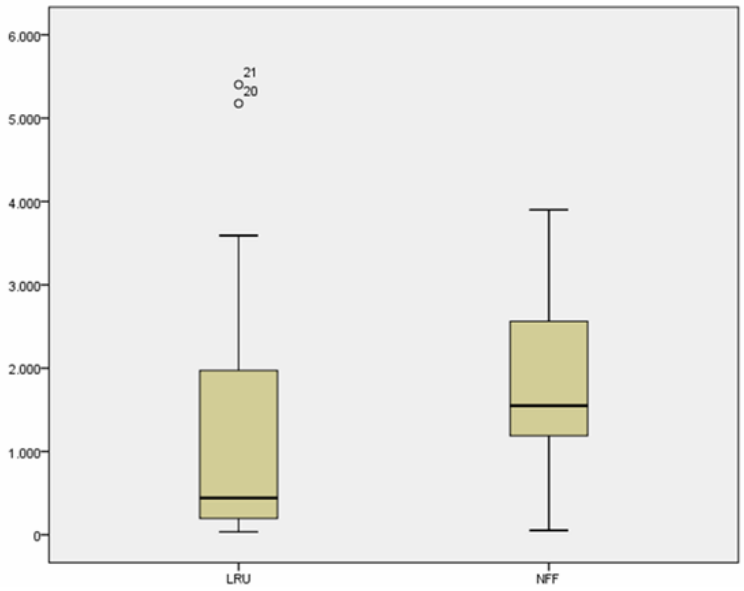


Figure 1. Comparison of boxplots for LRU and NFF categories

ADEQUACY OF WEIBULL DISTRIBUTION

Thus we now only consider LRU lifetime data and we conduct some statistical analysis in order to find an adequate model that fits the data well. In other words we are interested in determining the underlying distribution of failures concerning LRU. In the sequel we will first check whether a standard (two-parameter) Weibull model is adequate for our LRU data set. Our rationale stems from a remark from the introduction of [4], namely that " the Weibull model is the most popular, widely used model in reliability theory". Before proceeding with our experiment we provide some basic theory concerning these models. The c.d.f. (also sometimes called unreliability function) for Weibull takes the form

$F(t,\eta,\beta) = 1 - \exp[-\left(\frac{t}{\eta}\right)^{\beta}]$ with $t \geq 0$, and $\beta > 0$ is called shape parameter of the distribution whereas $\eta > 0$ is called scale parameter.

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The same argument as that provided in the introduction of [4] is then used which implies

linear relationship between $XX = \ln(tt)$ and $Y = \ln(-\ln(1 - FF)) = -\beta\beta \ln(\eta\eta) + \beta\beta\ln(tt)$, expressed by equation $Y = \beta X - \beta\beta\ln(\eta\eta)$, corresponding to relationship (4) in the aforementioned paper. Since FF is unobserved we follow the Median Rank approach and use

Benard's approximation (see [1]) in order to obtain the vertical coordinate for each point via the algebraic expression $\hat{F} = (jj - 0.3)/(NN + 0.4)$, where $\hat{F}$ is an estimator for the c.d.f. , j is the rank of the data point and N is the number of data points. This approach has been well documented in earlier literature (see [2], [4] and [5]); advantages of the method are reported by [2] who mentions that " it is deemed advantageous to routinely employ the median approximator when estimating the Weibull parameters". We then use the so- called Weibull plot technique consisting in plotting $\ln[-\ln(1 - FF)]$ versus $\ln(t)$. In case a Weibull distribution is adequate an approximate straight line is expected. Consequently, a linear regression approach is implemented thus providing a linear equation of the form $\hat{Y} = \hat{\beta}\hat{\beta}XX - \hat{\beta}\hat{\beta}\ln(\hat{\eta}\hat{\eta})$, sometimes also called the best fit line, where the parameter estimates $\beta\beta$ and $\eta\eta$ are computed using the ordinary least squares (OLS) method.

Using our LRU data a Weibull plot is shown in Figure 2, and the best fit line is also drawn on the same graph, which suggests a good fit of the straight line. Thus a two-parameter Weibull distribution seems reasonable for the LRU data set. The estimate of the shape parameter is the slope of the straight line $\hat{Y} = \hat{\beta}\hat{\beta}XX - \hat{\beta}\hat{\beta}\ln(\hat{\eta}\hat{\eta})$ and it is immediately provided by OLS. On the other hand the estimate $\hat{\eta}\hat{\eta}$ of the scale parameter can also be calculated from the results of OLS. Indeed results from OLS provide $\hat{\beta}\hat{\beta} = 0.73$ and $\hat{\beta}\hat{\beta}\ln(\hat{\eta}\hat{\eta}) = 5.19$ so that $\ln(\eta\eta) = 7.11$ and thus $\hat{\eta}\hat{\eta} = 1224$. Then, according to the best fit line (that is $\hat{YY} = 0.73XX - 5.19$) on the Weibull plot, $\hat{YY} = 0.0003$ for $XX = 7.11$. Note that since, according to Weibull distribution theory, the scale parameter always falls at the 63.2% point, one should expect this value of $\hat{YY}$ to be approximately equal to the value obtained for YY by taking $FF = 0.632$ in expression $YY = \ln(-\ln(1 - FF))$, which holds true because $YY = -0.00033$. Note that the estimated shape parameter $\hat{\beta}\hat{\beta} < 1$ indicates that the failure rate (or hazard rate)- which for Weibull distribution

is of the form $h(t, \eta, \beta) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}$ - decreases over time as LRU items are weeded out. This fact also agrees with our remark from Figure 1, namely that LRU items fail in general earlier than

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items from the NFF category. On the other hand estimators of the slope and intercept of the aforementioned best fit line are both highly significant with Student tt values equal to 18.85 for the former and -20.5 for the latter, and a value of 0.949 for the coefficient of determination R^2, thus corroborating the good fit suggested by the graphical methods.

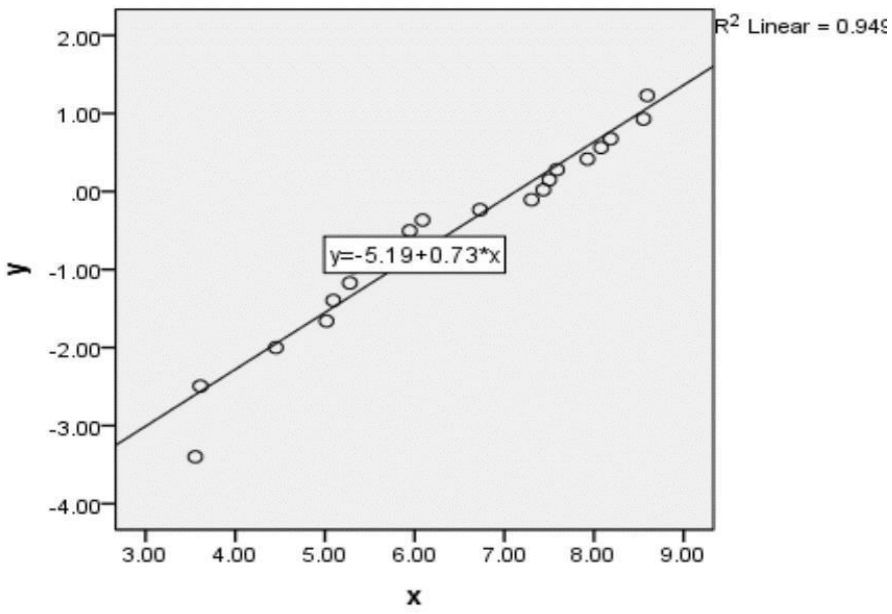


Figure 2 Weibull plot for LRU data

APPLICATIONS

We now provide two simple practical applications of the above results, which are useful for inferential purposes in an engineering context. The first one concerns the median of failure times following a Weibull distribution, which is of the form $(\ln(2))^{1/\beta\beta}$; letting $\eta\hat{\eta} = 1224$ and $\hat{\beta\beta} = 0.73$ the median takes the value 740.82. Thus half of LRU items are expected to fail (for the first time) within about 741 hours. The second application concerns the reliability function defined as $R(t,\eta,\beta) = 1 - F(t,\eta,\beta) = \exp[-\left(\frac{t}{\eta}\right)^{\beta}]$. Using this function one can estimate percentages of items that would still be alive at the end of some pre-specified time point. For example the approximate percentage of items still alive after 200 hours is given by (200, 1224, 0.73) = 0.766 , which amounts to 76.6% . The value 740.82 found for the median in the first application can be conceived as a special case of the second application for tt =740.82 since (740.82, 1224, 0.73) = 50%.

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CONCLUSION

According to our findings, LRU items tend to fail sooner, but NFF events have longer lives. If the assumption of a typical Weibull distribution for failure durations is correct (as it is in our instance), the value of the shape parameter should be smaller than one, suggesting that the number of LRU items declines with time. Our investigation has confirmed this finding. Furthermore, we believe that the considerable disparity in lives between our two groups is directly related to the fact that the form parameter value is far from one. A graphical analysis is done using a Weibull plot, which indicates a satisfactory match, and hence Weibull distribution theory may be used to describe LRU failure times. As seen in the preceding section, parameter estimation exposes essential properties of the underlying distribution and is thus very useful for practical applications. These findings may serve as a guidance for engineers and practitioners in the aerospace sector.

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