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A Study on B-Kenmotsu Manifold and Ricci Almost Solitons

Gurupadavva Ingalahalli [1], H.R. Mahadevaswamy[2], C.S. Bagewadi[3]

Department of Mathematics

J.N.N. College of Engineering, Navule, Shimoga - 577204, Karnataka, INDIA

Corresponding Authors' Email: - *gurupadavva@gmail.com*

Abstract

The purpose of the current paper is to study W_2-flat, ξ-W_2-flat, W_2-Ricci pseudo symmetric, W_2-Ricci semisymmetric, ϕ-W_2-semisymmetric β-Kenmotsu manifold and Ricci almost solitons.

Keywords: *Kenmotsu manifold, connection, Symmetric.*

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INTRODUCTION

Ricci solitons (g,U,ρ) are natural generalization of an Einstein metrics and is defined on a Riemannian manifold (M,g) by

$$(L_U g)(W,V) + 2S(W,V) + 2\rho g(W,V) = 0, \quad (1.1)$$

$\rho\rho$ is zero and expanding if $\rho\rho$ is positive respectively. Along with Mathematicians physicists also took lots of interest in the study of Ricci soliton equation in relation with the concept of string theory and also the fact that equation (1.1) is a special case of Einstein field equation. Ricci soliton is a special solution of the Ricci flow introduced by Hamilton [14] in the year 1982. In [25], R. Sharma studied Ricci solitons and obtained some interesting results in contact geometry. Thereafter Ricci solitons in contact metric manifolds have been studied by various authors such as M.M. Tripathi [29], A. Ghosh and R. Sharma [10], U.C. De and et. al. [5], H.G. Nagaraja and et. al. [20], C.S. Bagewadi and Gurupadavva Ingalahalli ([1], [2], [12], [13]), A.A. Shaikh and et. al. [4], S.K. Hui et. al. ([17], [18]), G.P. Pokhariyal and et. al. ([23]) and many others. In [24], Pigola et. al. introduced and studied some interesting results

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of Ricci almost soliton. Ricci almost soliton defined if the equation (1.1) exists for smooth function $\rho\rho$. Thereafter, R. Sharma [26], A. Ghosh ([8], [9]), S.K. Hui and et. al. ([15], [16]) also studied Ricci almost solitons. Later on many geometers studied Ricci almost solitons for different manifolds.

Kenmotsu [19] introduced and investigated the fundamental properties on the local structure of a new class of almost contact metric manifold which is called as Kenmotsu Manifold. In [21], Oubina introduced the notion of trans-Sasakian manifolds and are nearly related to locally conformal Kahler manifolds. A trans-Sasakian manifold is a generalization of both Kenmotsu and Sasakian manifolds. A trans-Sasakian manifold is of type (0,0) is called as cosympletic manifold, type of (α,0) is called as α -Sasakian and type of (0,β) is called as β-Kenmotsu manifold respectively. β-Kenmotsu manifolds have been studied by several authors. In [28], A.A. Shaikh and S.K. Hui studied some properties of β-Kenmotsu manifolds.

Inspired by the above research work, in the present paper we investigated the geometric nature of the Ricci almost solitons in β-Kenmotsu manifolds. Further, we studied W_2-flat, ξ-W_2-flat, W_2-Ricci pseudosymmetric, W_2-Ricci semisymmetric, ϕ-W_2-semisymmetric β-Kenmotsu manifold and Ricci almost solitons.

2. Basic Concepts of β-Kenmotsu manifold

An odd dimensional smooth manifold M is called an almost contact metric manifold [3] if it admits a (1,1)-tensor field ϕ, a vector field ξ, a 1-form η and a Riemannian metric g, which satisfy

$$\phi^2 W = -W+\eta(W)\xi, \quad \phi\xi = 0, \eta(\xi) = 1, \eta(\phi W) = 0, \tag{2.1}$$

$$g(\phi W,\phi V) = g(W,V)-\eta(W)\eta(V), \eta(W) = g(W,\xi), g(\phi W,V) = -g(W,\phi V), \tag{2.2}$$

for all vector fields W,V. An almost contact metric manifold (M,ϕ,ξ,η,g) is said to be a β-Kenmotsu manifold if the following conditions hold [28]:

$$\nabla_U \xi = \beta[U-\eta(U)\xi], \tag{2.3}$$

$$(\nabla_U \phi)V = \beta[g(\phi U,V)\xi-\eta(V)\phi U], \tag{2.4}$$

$$(\nabla_U \eta)(V) = \beta[g(U,V)-\eta(U)\eta(V)]. \tag{2.5}$$

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In particular, if $\beta\beta = 1$ then the $\beta\beta$-Kenmotsu manifold is called as Kenmotsu manifold [19]. If

$\beta\beta = ccoonnsstt$ then the $\beta\beta$-Kenmotsu manifold is called as homothetic Kenmotsu manifold.

The following relations holds for a $\beta\beta$-Kenmotsu manifold [28]:

$R(U,V)\xi=-\beta^2 [\eta(V)U-\eta(U)V]+ (U\beta)[V-\eta(V)\xi]- (V\beta)[U-\eta(U)\xi],$ 2.6

$R(\xi,U)V=(\beta^2+(\xi\beta))[\eta(V)U-g(U,V)\xi],$ 2.7

$\eta(R(U,V)Z)=\beta^2 [\eta(V)g(U,Z)-\eta(U)g(V,Z)]- (U\beta)[g(V,Z)-\eta(V)\eta(Z)] + (V\beta)[g(U,Z)-\eta(U)\eta(Z)],$ 2.8

$S(U,\xi)=-(2n\beta^2+(\xi\beta))\eta(U)- (2n-1)(U\beta).$ 2.9

for all vector fields $U,V,Z \in \chi(M)$.

3. Ricci almost solitons on β-Kenmotsu manifold

In this section, we study some properties of Ricci almost solitons on β-Kenmotsu manifold: Let us consider (g,ξ,ρ) be a Ricci almost soliton in a β-Kenmotsu manifold M. By inserting $U=\xi$ in (1.1), we have

(3.1) $2S(W,V)=-(L_\xi g)(W,V)- 2\rho g(W,V).$

With the help of (2.3), we get

$(L_\xi g)(W,V)= g(\nabla_W \xi,V)+ g(W,\nabla_V \xi)$ 3.2
$= 2\beta[g(W,V)-\eta(W)\eta(V)].$

From (3.1) and (3.2), we have

$S(W,V)= -(\rho + \beta)g(W,V)+ \beta\eta(W)\eta(V).$ 3.3

Now, we can state the following statement

Theorem 3.1. A β-Kenmotsu manifold admitting Ricci almost soliton is an η-Einstein manifold. From (3.3), we get

$QV = -(\rho + \beta)+ \beta\eta(V)\xi,$ 3.4

$S(V,\xi)=-\rho\eta(V),$ 3.5

$r = -(2n + 1)\rho-2n\beta,$ 3.6

where QQ is the Ricci operator defined as $gg(QQFF,GG) = SS(FF,GG)$ for all vector fields FF,GG and rr is the scalar curvature of MM.

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4. W_2-flat β-Kenmotsu manifold and Ricci almost solitons

This section is dedicated to the study of W_2-flat β-Kenmotsu manifold and Ricci almost solitons. In [22], Pokhariyal and Mishra introduced and studied some properties of the curvature tensor called as the W_2-curvature tensor. W_2-curvature tensor is defined as

$$W_2(G,H)I = R(G,H)I + 1/2n\,[g(G,I)QH - g(H,I)QG], \quad 4.1$$

for all vector fields G,H,I on M and R is the curvature tensor and Q is the Ricci operator.

A β-Kenmotsu manifold is said to be W_2-flat if

$$W_2(G,H)I = 0. \quad 4.2$$

From (4.1) and (4.2), we get

$$R(G,H)I + 1/2n\,[g(G,I)QH - g(H,I)QG] = 0 \quad 4.3$$

By taking an innerproduct with ξ in (4.3), we obtain

$$g(R(G,H)I,\xi) + 1/2n\,[g(G,I)g(QH,\xi) - g(H,I)g(QG,\xi)] = 0. \quad 4.4$$

By using (2.8), (3.3), (3.5) in (4.4) and on simplification, we have

$$[\beta^2 - \rho/2n][g(G,I)\eta(H) - g(H,I)\eta(G)] - (G\beta)[g(H,I) - \eta(H)\eta(I)] + (H\beta)[g(G,I) - \eta(G)\eta(I)] = 0. \quad 4.5$$

Putting $G=\xi$ in (4.5) and on simplification, we get

$$[\beta^2 - \rho/2n + (\xi\beta)][\eta(G)\eta(I) - g(G,I)] = 0 \quad 4.6$$

which implies that,

$$\rho = 2n[\beta^2 + (\xi\beta)]. \quad 4.7$$

Hence, we state the following theorem:

Theorem 4.2. If (g,ξ,ρ) is a Ricci almost soliton on a W_2-flat β-Kenmotsu manifold, then the Ricci almost soliton is always expanding.

Corollary 4.1. If (g,ξ,ρ) is a Ricci almost soliton on a W_2-flat Kenmotsu manifold, then the Ricci almost soliton is always expanding.

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5. ξ-W_2-flat β-Kenmotsu manifold and Ricci almost solitons

In this section, we study Ricci almost solitons in a ξ-W_2-flat β-Kenmotsu manifold:

A β-Kenmotsu manifold is said to be ξ-W_2-flat if

$$W_2(G,H)\xi = 0 \quad (5.1)$$

for any vector fields GG, HH on MM. From (4.1) and (5.1), we get

$$R(G,H)\xi + 1/2n[g(G,\xi)QH - g(H,\xi)QG] = 0. \quad (5.2)$$

By using (2.6), (3.5) in (5.2) and on simplification, we have

$$[-\beta^2+(\rho+\beta)/2n][\eta(H)G-\eta(G)H]+(G\beta)[H-\eta(H)\xi]-(H\beta)[G-\eta(G)\xi] = 0. \quad (5.3)$$

Putting $HH = \xi\xi$ in (5.3) and on simplification, we get

$$[-\beta^2+(\rho+\beta)/2n-(\xi\beta)][G-\eta(G)\xi]=0 \quad (5.4)$$

which implies that,

$$\rho=2n[\beta^2+(\xi\beta)]-\beta. \quad (5.5)$$

Hence, we state the following theorem:

Theorem 5.3. If (g,ξ,ρ) is a Ricci almost soliton on a ξ-W_2-flat β-Kenmotsu manifold, then the Ricci almost soliton is expanding.

Corollary 5.2. If (g,ξ,ρ) is a Ricci almost soliton on a ξ-W_2-flat Kenmotsu manifold, then the Ricci almost soliton is expanding.

6. W_2-Ricci pseudo-symmetric β-Kenmotsu manifold and Ricci almost solitons

In this section, we study Ricci almost solitons on W_2-Ricci pseudosymmetric β-Kenmotsu manifold:

In ([6], [7]), Deszcz initiated the study of Ricci pseudo-symmetric manifold. In [11], Jahanara and et.al. studied the geometrical properties of the Ricci pseudo-symmetric manifolds in the Riemannian case. A β-Kenmotsu manifold (M,g) is said to be Ricci pseudo-symmetric ([6], [7]) if the tensor R · S and Q(g,S) are linearly dependent, where Q(g,S) is the Tachibana tensor

$$(R(G,H)\cdot S)(I,K) = -S(R(G,H)I,K) - S(I,R(G,H)K), \quad (6.1)$$

$$Q(g,S)(I,K;G,H) = -S((G \wedge g H)I,K) - S(I,(G \wedge g H)K), \quad (6.2)$$

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and

$$(G \Lambda g H)I = g(H,I)G - g(G,I)H \quad 6.3$$

for all vector fields $G,H,I,K \in \chi(M)$, R is the curvature tensor of M. Then (M,g) is Ricci pseudo-symmetric if and only if

$$(R(G,H)\cdot S)(I,K) = L_S\, Q(g,S)(I,K;G,H) \quad 6.4$$

holds on $UUSS = xx \in MM: SS - rr2nn+1\ gg \neq 0\ aatt\ xx$, for some function $LLSS$ on $UUSS$. If $RR \cdot SS = 0$, then MM is said to Ricci semi-symmetric. Every Ricci semi-symmetric manifold is Ricci pseudo-symmetric but the converse is not true [7]. In ([17], [27]), A.A. Shaikh and Hui et. al studied and obtained some interesting results in Ricci pseudo-symmetric generalized quasi-Einstein manifolds.

A $WW2$-Ricci pseudo-symmetric $\beta\beta$-Kenmotsu manifold $(MM,)$ is defined as

$$(W_2(G,H)\cdot S)(I,K) = L_S\, Q(g,S)(I,K;G,H) \quad 6.5$$

holds on $UUSS = xx \in MM: SS - rr2nn+1\ gg \neq 0\ aatt\ xx$, where $LLSS$ is some function on $UUSS$. Let us consider a $WW2$-Ricci pseudo-symmetric $\beta\beta$-Kenmotsu manifold whose metric is Ricci almost soliton. Then by virtue of (6.5), we have

$$S(W_2(G,H)I,K) + S(I,W_2(G,H)K) = L_S\,[g(H,I)S(G,K) - g(G,I)S(H,K) + g(H,K)S(G,I) - g(G,K)S(H,I)] \quad 6.6$$

Putting $= \xi\xi$ in (6.6) and by virtue of (3.3) and on simplification, we get

$$-\beta\beta + \rho\rho gg WW2GG,, + \beta\beta\eta\eta WW2GG,HH\xi\xi\eta\eta KK - \rho\rho\eta\eta(WW2(GG,HH)KK) \quad 6.7$$

$$= \beta\beta LLSSggHH,KK\eta\eta GG - ggGG,KK\eta\eta HH.$$

From (4.1), we get

$$W_2(G,H)\xi = -\beta^2\,[\eta(H)G - \eta(G)H] + (G\beta)[H - \eta(H)\xi] - (H\beta)[G - \eta(G)\xi] + 1/2n\,[\eta(G)QH - \eta(H)QG]. \quad 6.8$$

By using (6.8) in (6.7), we obtain

$$\beta[\beta^2 - \rho/n - \beta/2n][\{g(G,K)\eta(H) - g(H,K)\eta(G)\} - \beta(G\beta)[g(H,K) - \eta(K)\eta(H)] + \beta(H\beta)[g(G,K) - \eta(K)\eta(G)]] = \beta L_S\,[g(H,K)\eta(G) - g(G,K)\eta(H)]. \quad 6.9$$

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Putting $HH = \xi\xi$ in (6.9) and on simplification, we get

$$[L_S-\beta/2n-\rho/n+(\beta^2+\xi\beta)][g(G,K)-\eta(G)\eta(K)] = 0 \quad 6.10$$

for all vector fields GG and KK, which follows that

$$L_S=\beta/2n+\rho/n-(\beta^2+\xi\beta). \quad 6.11$$

This leads to the following:

Theorem 6.4. If (g,ξ,ρ) is a Ricci almost soliton on a W_2-Ricci pseudosymmetric β-Kenmotsu manifold, then $L_S=\beta/2n+\rho/n-(\beta^2+\xi\beta)$.

Corollary 6.3. If (g,ξ,ρ) is a Ricci almost soliton on a W_2-Ricci pseudosymmetric Kenmotsu manifold, then the Ricci almost soliton is expanding, shrinking and steady for all $\rho > 0, \rho < 0, \rho = 0$.

7. W_2-Ricci semisymmetric β-Kenmotsu manifold and Ricci almost solitons

In this section we study Ricci almost solitons on W_2-Ricci semisymmetric β-Kenmotsu manifold that is

$W_2(\xi,G)\cdot S = 0$ which gives

$$S(W_2(\xi,G)H,I) + S(H,W_2(\xi,G)I) = 0. \quad 7.1$$

By using (3.3) in (7.1), we get

$$-(\beta+\rho)[g(W_2(\xi,G)H,I)+g(H,W_2(\xi,G)I)] \quad 7.2$$
$$+\beta[\eta(W_2(\xi,G)H)\eta(I)+\eta(H)\eta(W_2(\xi,G)I)]=0.$$

From (4.1), we get

$$W_2(\xi,H)I = (\beta^2 + \xi\beta)[\eta(I)H - g(H,I)\xi]+1/2n\,[\eta(I)QH-g(H,I)Q\xi]. \quad 7.3$$

By using (7.3) in (7.2) and on simplification, we obtain

$$-((\beta+\rho))/2n[S(G,I)\eta(H) + S(G,H)\eta(I) + \rho\{g(G,I)\eta(H)+ g(G,H)\eta(I)\}] \quad 7.4$$
$$+\beta[(\beta^2+\xi\beta)-\rho/2n]\{2\eta(G)\eta(H)\eta(I)- g(G,H)\eta(I)-g(G,I)\eta(H)\} = 0.$$

By virtue of (3.3) in (7.4) and on simplification, we get

$$[(\beta(\beta+\rho))/2n-\beta(\beta^2+\xi\beta)+\beta\rho/2n]\{g(G,H)\eta(I)+ g(G,I)\eta(H)- 2\eta(G)\eta(H)\eta(I)\} = 0. \quad 7.5$$

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The above equation (7.5) implies that

$[(\beta(\beta+\rho))/2n-\beta(\beta^2+\xi\beta)+\beta\rho/2n]=0.$ 7.6

From (7.6), we get

$\rho=(2n(\beta^2+\xi\beta) -\beta)/2$ 7.7

Hence, we state the following

Theorem 7.5. If (g,ξ,ρ) is a Ricci almost soliton on a W_2-Ricci semisymmetric β-Kenmotsu manifold, then the Ricci almost soliton is always expanding.

Corollary 7.4. If (g,ξ,ρ) is a Ricci almost soliton on a W_2-Ricci semisymmetric Kenmotsu manifold, then the Ricci almost soliton is expanding.

8. ϕ-W_2-semisymmetric β-Kenmotsu manifold and Ricci almost solitons

In this section, we study Ricci almost solitons on ϕ-W_2-semisymmetric β-Kenmotsu manifold that is

$W_2 \cdot \phi = 0$ which gives

$W_2 (G,H)\phi I-\phi(W2(G,H)I) = 0.$ 8.1

Putting $II=\xi\xi$ in (8.1) and by virtue of (4.1), we get

$\phi(W_2 (G,H)\xi) = 0.$ 8.2

By using (6.8) in (8.2), we have

$-\beta^2 [\eta(H)\phi G-\eta(G)\phi H]+(G\beta)\phi H-(H\beta)\phi G+1/2n[\eta(G)\phi QH-\eta(H)\phi QG] = 0.$ 8.3

By using (3.4) in (8.3) and on simplification, we get

$[((\beta+\rho))/2n-\beta^2]\{\eta(H)\phi G-\eta(G)\phi H\}+ (G\beta)\phi H-(H\beta)\phi G = 0.$ 8.4

Putting $G=\xi$ in (8.4), we obtain

$[((\beta+\rho))/2n-\beta^2-(\xi\beta)]\phi H = 0.$ 8.5

which implies

$\rho= 2n(\beta^2+\xi\beta)-\beta.$ 8.6

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Hence, we state the following

Theorem 8.6. If (g,ξ,ρ) is a Ricci almost soliton on a ϕ-W_2-Ricci semisymmetric β-Kenmotsu manifold, then the Ricci almost soliton is expanding.

Corollary 8.5. If (g,ξ,ρ) is a Ricci almost soliton on a ϕ-W_2-Ricci semisymmetric Kenmotsu manifold, then the Ricci almost soliton is expanding.

CONCLUSION

The geometry of Ricci solitons, has been the main target of attention of the many mathematicians. The study of Ricci soliton is of great importance with in the areas of Riemannian geometry, differential geometry and in physics also. Particularly, it's become more important after Perelman applied Ricci solitons to solve the long standing Poincare conjecture posed in 1904. Ricci solitons are the special solutions of the Ricci flow introduced by Hamilton in the year 1982. Ricci soliton generalizes the notion of Einstein metric on a Riemannian manifold. Theoretical physicists have also been taking interest in the study of Ricci soliton in relation with string theory and the fact that Ricci soliton equation is a special case of Einstein field equations. Ricci soliton is said to be shrinking, steady, and expanding according as ρ is negative, zero and positive respectively. The Ricci almost soliton are often considered as a generalization of Einstein metric as well as Ricci soliton with smooth function ρ. In this paper we study the geometric nature of the Ricci almost solitons in β-Kenmotsu manifold with W_2-flat, ξ-W_2-flat, W_2-Ricci pseudosymmetric, W_2-Ricci semisymmetric and ϕ-W_2-semisymmetric conditions which is expanding with variable ρ.

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