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Complement Operator & K – Horizontal Symmetric Measurable Function

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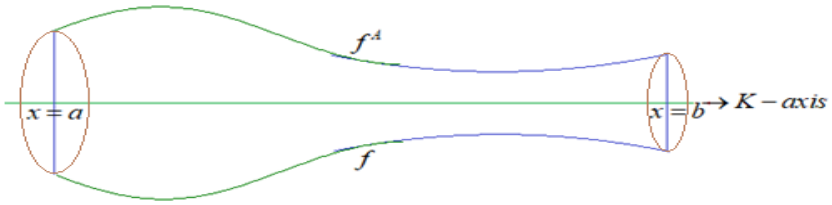
Abstract

Measure and probability distribution are closely related in the real time situations. For a measurable function, A system of measurable functions forming a hollow beam about the K – axis that is parallel to the X – axis formed by rotating through an angle 2π. if the given function f is a function of bounded variation, then every function forming this beam is a generator of the beam and is a function of bounded variation. the projection of the beam upon the plane having K – axis as the axis of symmetry will be a symmetric function f^A and these two functions will enclose a symmetric region between the vertical limits x = a and x = b. in the previous paper titled "K – Horizontal Symmetric Measurable functions, the function f^A is noted to be the K – Horizontal Symmetric Measurable function of f. now, it is termed as the adjoint of the function f. From the above discussion, f^A is also a generator of the hollow beam and is a function of bounded variation whenever $f \in BV$. From the properties of the functions of bounded variation, we have $P_a^b(f) - N_a^b(f) = f(b) - f(a)$ and consequently, about adjoint function, it follows $P_a^b(f^A) - N_a^b(f^A) = K + f(a) - f(b)$

Keywords: *Complement Operator, Adjoint function, Measurable Function, X-axis*

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INTRODUCTION



A function f is measurable if $D=\{x:f(x)>\alpha,\alpha\in\mathbf{R}\}$ or $D=\{x:f(x)\le\alpha,\alpha\in\mathbf{R}\}$ is m^*-measurable. a measurable function may be increasing or decreasing or both on a given domain $[a,b]$. So, a partition $\{a=x_0<x_1<x_2<...<x_k=b,x_i\in\mathbf{R},0\le i\le k\}$ on $[a,b]$ can be taken such that f is monotonic on each of $[x_{i-1},x_i],1\le i\le k$. In the present paper, we consider the **non negative** measurable function f and then extend the results to general measurable functions.

If f is increasing on $[x_{i-1},x_i]$, then $f(x_i)-f(x_{i-1})\ge 0$ and so, we add $f(x_i)-f(x_{i-1})$ to $p_a^b(f)$ and otherwise, we add $f(x_i)-f(x_{i-1})$ to $n_a^b(f)$. We can construct infinitely many partitions on [a, b] and so, infinite pairs $p_a^b(f)\&\,n_a^b(f)$ are possible.

$$p_a^b(f)=\sum_{i=1}^{k}\left[f(x_i)-f(x_{i-1})\right]^+ \;\&\; n_a^b(f)=\sum_{i=1}^{k}\left[f(x_i)-f(x_{i-1})\right]^-$$

See that $p_{x_{i-1}}^{x_i}(f)$ takes non zero on the sub interval $[x_{i-1},x_i]$ where $n_a^b(f)$ takes 0 and vice – versa. So, $p_a^b(f)-n_a^b(f)=\sum_{j=1}^{k}\{f(x_j)-f(x_{j-1})\}$

Chapter 1

Definition 1.1: a function f defined on a domain [a, b] is increasing if it is increasing in every neighbourhood in [a, b]. The function f is increasing in the neighbourhood $(x-\delta,x+\delta)$ if $x_1,x_2\in(x-\delta,x+\delta),x_1<x_2\Rightarrow f(x_1)\le f(x_2)$. Otherwise the function is decreasing.

Note 1.1: one can construct a subdivision of [a, b] as $\{a=x_0<x_1<...<x_k=b\}$ such that on each subinterval $[x_{i-1},x_i],1\le i\le k$, the function f is monotonic. There can be infinitely many

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such subdivisions on [*a*, *b*].

Definition 1.2: a function *f* increasing on $[x_1, x_2]$, then *f* is said to have positive variation on $[x_1, x_2]$. Otherwise it is having the negative variation.

Definition 1.3: the m^*-measure of *f* is $|M-m|$ where $M = \sup f$ on $[a,b]$ and $m = \inf f$ on $[a,b]$.

Note 1.2: a function *f* on [*a*, *b*] may be increasing or decreasing or both. Also, if *f* is increasing on $[x_{i-1}, x_i]$, then it is not necessary that *f* increases on $[x_i, x_{i+1}]$ and vice versa. There are infinite subdivisions on [*a*, *b*] and so, with reference to each subdivision, there is one positive variation $p_a^b(f)$ and one negative variation $n_a^b(f)$ are possible.

Definition 1.4: The positive variation of *f* on [*a*, *b*] is the sum of the positive variations on those neighbourhoods where *f* is increasing. It is denoted by $p_a^b = \sum_i p_{x_{i-1}}^{x_i} = \sum_i \lfloor f(x_i) - f(x_{i-1}) \rfloor_+$

Definition 1.5: the negative variation of *f* on [*a*, *b*] is the sum of the negative variations of those neighbourhoods where *f* is decreasing. It is denoted by $n_a^b = \sum_j n_{x_{j-1}}^{x_j} = \sum_j \lceil f(x_j) - f(x_{j-1}) \rceil_-$

Definition 1.6: $\sup p_a^b(f) = P_a^b(f)$ and $\sup n_a^b(f) = N_a^b(f)$

Note 1.3: with reference to each subdivision of [*a*, *b*], the difference of positive and negative variation remains constant and is equal to $f(b) - f(a)$.

That is, $f(b) - f(a) = p_a^b(f) - n_a^b(f)$

While this equality is true for every pair of positive and negative variations, it is true for their respective suprema also. That is, $f(b) - f(a) = P_a^b(f) - N_a^b(f)$

Definition 1.7: with reference to each subdivision, the total variation of *f* on [*a*, *b*] is $p_a^b(f) + n_a^b(f) = t_a^b(f) = \sum_{i=1}^k f(x_i) - f(x_{i-1})$. The definition of total variation holds with

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Definition 1.8: if $T_a^b(f) < \infty$, then we say f is a function of bounded variation on [a, b] denoted by $f \in BV[a,b]$

Chapter 2

If f is a measurable function on D, then f^{KS} is the symmetric function of f about the horizontal axis K which is also measurable with the domain D.

Definition 2.1: $p_a^b(f^{KS}) = 2K - \sum_{j=1}^{k} \lceil f(x_j) - f(x_{j-1}) \rceil_+$ and $n_a^b(f^{KS}) = 2K - \sum_{i=1}^{k} \lceil f(x_i) - f(x_{i-1}) \rceil_-$

Note 2.1: while there are infinitely types subdivisions on [a, b], with reference to each subdivision, $f(b) - f(a) = p_a^b(f) - n_a^b(f)$, it follows that $p_a^b(f^{KS}) - n_a^b(f^{KS}) = f(a) - f(b)$ a constant. This result is true for the suprema case also. Consequently, $f(b) - f(a) = P_a^b(f) - N_a^b(f)$ and $T_a^b(f^{KS}) = P_a^b(f^{KS}) - N_a^b(f^{KS})$

Theorem 2.1

if f is a function of bounded variation on [a, b], then f^{KS} is also a function of bounded variation with a total variation $T_a^b(f^{KS}) = 2K - T_a^b(f)$

Proof: $T_a^b(f) = s < \infty$ and $2K$ is a finite real number implies $2K - s < \infty$

$\therefore 2K - T_a^b(f) < \infty$

Chapter 3

Definition 3.1: if $f \in BV$, then $\|f\| = \sup \sum_{i=1}^{k} \int_{x_{i-1}}^{x_i} |f(x_i) - f(x_{i-1})|$ where

$\{a = x_0 < x_1 < \ldots < x_k = b\}$ is an arbitrary partition on [a, b].

Definition 3.2: if f is a function of bounded variation on [a, b], then the adjoint of f denoted by f^A is a function that is symmetric to f about a horizontal axis K on [a, b].

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Theorem 3.1:

for every $f \in BV$, there exists a unique $f^{KS} \in BV$ such that
$T_a^b\left(f^{KS}\right) = {}^2K - T_a^b(f)$

Definition 3.3: K – symmetric conjugation is the operation $2K - \langle \alpha f, \alpha \rangle = \overline{\langle \alpha f, \alpha \rangle}$

Definition 3.4: if $f \in BV$ and m^* – measurable with the domain D, then for every pair $(\alpha, \beta) \in D \times D$, $g : D \times D \to \sqcup$ defined by $g_\beta(\alpha) = \langle \alpha f, \beta \rangle$ is called the inner product that satisfies

3.1.1. $g_\alpha(\alpha) = \langle \alpha f, \alpha \rangle \geq 0$

3.1.2. $\bar{g}_\beta(\alpha) = \overline{\langle \alpha f, \alpha \rangle} = \langle \alpha, \alpha f^{KS} \rangle$

3.1.3. $g_\alpha(\alpha) = \langle \alpha f, \alpha \rangle = 0$ if and only if $\alpha = 0$ a real number

3.1.4. $g_\gamma(\alpha + \beta) = \langle (\alpha + \beta) f, \gamma \rangle = \langle \alpha f, \gamma \rangle + \langle \beta f, \gamma \rangle$

Note 3.1: the notation $\langle \alpha f, \alpha \rangle$ stands for K – symmetric conjugation.

Theorem 3.2

if f is a function of bounded variation on [a, b], then

of f.

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