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Real-Time Control with Uncertainty: A Comprehensive Review

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Abstract

Real-time control systems are increasingly deployed in complex and dynamic environments such as autonomous vehicles, smart grids, robotics, aerospace systems, and biomedical devices. A defining challenge in such systems is the presence of uncertainty arising from modeling errors, external disturbances, sensor noise, time delays, and unpredictable environmental changes. This paper presents a detailed review of real-time control under uncertainty, focusing on theoretical foundations, mathematical modeling, uncertainty representation, and modern control strategies. Classical approaches such as robust control and adaptive control are discussed alongside stochastic control, model predictive control, and data-driven methods. Special emphasis is placed on uncertainty quantification, real-time computational constraints, and stability considerations. The paper also highlights recent trends and practical challenges in implementing uncertainty-aware controllers in real-time environments. Tables and conceptual figures are included to summarize methods and compare their strengths and limitations. The review aims to provide a structured understanding of real-time control with uncertainty for researchers and practitioners in engineering and applied mathematics.

Keywords: *Real-time control, uncertainty modeling, robust control, stochastic systems, adaptive control, uncertainty quantification, feedback systems*

INTRODUCTION

Real-time control refers to the process of computing control actions within strict time constraints, where delayed decisions may lead to system degradation or failure. In practical

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systems, control decisions must be made continuously based on real-time measurements, while accounting for uncertainties that affect system behavior. These uncertainties may originate from incomplete system models, unknown parameters, disturbances, or noisy measurements. Unlike offline control design, real-time control must operate with limited computational resources and incomplete information.

Uncertainty is unavoidable in real-world systems. Even well-modeled physical processes are subject to parameter variations, aging effects, and environmental disturbances. In real-time applications such as flight control, power system regulation, or robotic manipulation, ignoring uncertainty can result in instability or unsafe operation. Therefore, modern control theory has evolved to explicitly address uncertainty, balancing performance, robustness, and computational efficiency.

This review paper focuses on the intersection of real-time control and uncertainty. The objective is to present a coherent overview of modeling techniques, control strategies, and implementation challenges. While the field is broad, the discussion emphasizes methods that are suitable for real-time deployment, where fast computation and reliability are critical.

= SOURCES AND TYPES OF UNCERTAINTY IN CONTROL SYSTEMS

Uncertainty is an inherent characteristic of real-world control systems and arises from multiple interacting factors. In practical implementations, it is rarely possible to obtain a perfectly accurate mathematical description of a system or its operating environment. As a result, uncertainty must be explicitly considered during controller design, especially in real-time applications where rapid decision-making is required. Broadly, uncertainties in control systems can be classified based on their source, structure, and mathematical representation. The most common types include model uncertainty, external disturbances, measurement noise, and time-delay-related uncertainty.

Model Uncertainty

Model uncertainty refers to discrepancies between the actual physical system and the mathematical model used for analysis and controller synthesis. In most engineering problems, system models are derived using simplifying assumptions to reduce complexity. While such

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approximations are necessary, they often lead to inaccuracies that become significant during real-time operation.

One major source of model uncertainty is **parametric uncertainty**, where system parameters such as mass, stiffness, damping, resistance, or friction coefficients are not known exactly or vary with time. For instance, in robotic manipulators, payload variations directly affect inertia parameters, while in electrical systems, component aging can change resistance or inductance values. Similarly, aerodynamic coefficients in flight control systems depend on operating conditions and are difficult to estimate precisely.

Another important aspect is **unmodeled or neglected dynamics**. High-frequency dynamics, nonlinear effects, or coupling terms are sometimes omitted to simplify control design. However, these neglected dynamics can become dominant under certain operating conditions, leading to performance degradation or instability. In real-time control, model uncertainty is particularly challenging because there is limited time to identify or compensate for such discrepancies online.

To handle model uncertainty, control strategies such as robust control and adaptive control are often employed. Robust control assumes bounded uncertainty and guarantees stability within specified limits, while adaptive control attempts to adjust model parameters during operation. Despite these approaches, model uncertainty remains one of the most critical challenges in real-time control design.

External Disturbances

External disturbances are unwanted inputs that affect the system but are not directly measured or controlled. Unlike model uncertainty, which is internal to the system description, disturbances typically originate from the environment in which the system operates. These disturbances are often unpredictable, time-varying, and may have complex statistical properties.

Common examples include wind gusts acting on aircraft and drones, load variations in power systems, road irregularities affecting vehicle suspension systems, and contact forces in robotic manipulation tasks. In industrial processes, disturbances may arise from fluctuations in raw

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material properties or ambient temperature changes. Such disturbances can significantly influence system behavior, especially in sensitive or lightly damped systems.

In real-time control, disturbances pose a serious challenge because their effects must be counteracted immediately to maintain stability and performance. Feedforward compensation is effective only when disturbances can be measured or predicted, which is rarely the case. As a result, feedback control plays a crucial role in disturbance rejection.

From a modeling perspective, disturbances are often treated as unknown inputs with bounded energy or stochastic properties. In many practical systems, disturbance characteristics change over time, making fixed disturbance models insufficient. This motivates the use of disturbance observers, stochastic control methods, and robust control techniques that explicitly account for worst-case or probabilistic disturbance behavior.

Measurement Noise

Measurement noise arises from imperfections in sensors and data acquisition systems used to monitor system states and outputs. In real-time control, sensor data forms the basis for feedback decisions, and any uncertainty in measurements directly affects control performance.

Sources of measurement noise include electronic noise, quantization effects due to finite sensor resolution, thermal fluctuations, electromagnetic interference, and sensor drift. For example, encoders used for position measurement may suffer from quantization errors, while accelerometers are sensitive to high-frequency noise. Over time, sensors may also develop biases or calibration errors.

Measurement noise is particularly problematic because it can lead to incorrect state estimation. Controllers that react strongly to noisy measurements may generate excessive control actions, causing actuator wear or instability. This issue is amplified in high-gain feedback systems and fast real-time controllers.

To mitigate the effects of measurement noise, filtering and estimation techniques such as Kalman filters, extended Kalman filters, and observers are widely used. These methods attempt to separate true system behavior from noise by exploiting system dynamics and statistical

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assumptions. However, filtering introduces additional computational load and may cause delays, which must be carefully managed in real-time applications.

Time Delays and Computational Uncertainty

Time delays are an unavoidable source of uncertainty in real-time control systems. Delays may occur during sensing, data transmission, computation, or actuation. In modern control architectures, especially networked and distributed systems, communication delays can vary unpredictably due to network congestion or packet loss.

Sensing delays occur when measurements are not available instantaneously, while actuation delays arise from actuator dynamics or signal processing. Computational delays result from the time required to execute control algorithms, which may vary depending on processor load or algorithm complexity. These delays effectively introduce uncertainty in the timing of control actions.

Time-delay-related uncertainty can significantly degrade system performance and may even destabilize an otherwise well-designed controller. In fast systems such as power electronics or robotic systems, even small delays can have a large impact. Moreover, variable or random delays are more difficult to handle than constant delays, as they cannot be easily compensated. To address time-delay uncertainty, control designers use techniques such as delay compensation, predictive control, and time-stamped feedback. Real-time scheduling and computational optimization also play a crucial role in reducing uncertainty due to delays. Nevertheless, ensuring reliable performance in the presence of time-varying delays remains an active research area.

Table 1: Common Sources of Uncertainty in Real-Time Control Systems

Source of Uncertainty	Description	Example
Model uncertainty	Inaccurate or incomplete system models	Unknown parameters
External disturbances	Unmeasured environmental effects	Wind, load changes

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Source of Uncertainty	Description	Example
Measurement noise	Sensor inaccuracies	Encoder noise
Time delays	Variable computation or communication delays	Networked control

MATHEMATICAL REPRESENTATION OF UNCERTAINTY

The mathematical representation of uncertainty plays a central role in the analysis and design of control systems operating in real time. The choice of uncertainty model directly influences controller structure, stability guarantees, and computational requirements. Since uncertainty can arise from different physical and environmental sources, no single representation is universally applicable. Instead, several modeling approaches have been developed, each suited to particular types of uncertainty and levels of available information. This section reviews the most commonly used representations in control theory.

Deterministic Bounded Uncertainty

Deterministic bounded uncertainty assumes that uncertain parameters or disturbances lie within known bounds. In this framework, uncertainty is not treated as random but as an unknown quantity constrained within a specified interval or region. For example, a system parameter such as mass or damping may be assumed to vary within a known range based on design tolerances or experimental data.

Mathematically, bounded uncertainty is often represented using inequalities. A system parameter ppp may be expressed as

p∈[pmin,pmax]p \in [p_{\min}, p_{\max}]p∈[pmin,pmax]

or equivalently as a nominal value plus a bounded perturbation. Similarly, disturbances may be assumed to satisfy energy or magnitude bounds.

This representation is widely used in robust control, where the objective is to guarantee stability and performance for all admissible uncertainties. Techniques such as H-infinity control and μ-synthesis are based on this idea and aim to minimize the worst-case effect of uncertainty on system behavior.

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The main advantage of deterministic bounded models is their simplicity and strong stability guarantees. However, they often lead to conservative controller designs, as the controller must be robust against extreme and possibly unlikely scenarios. In real-time applications, this conservatism may reduce performance under normal operating conditions.

Stochastic Uncertainty

Stochastic uncertainty models uncertainty as a random process characterized by probability distributions. This approach is appropriate when uncertainty arises from random phenomena such as sensor noise, environmental disturbances, or unmodeled dynamics with statistical properties. Unlike deterministic models, stochastic representations allow the designer to consider average performance rather than worst-case behavior.

System dynamics with stochastic uncertainty are commonly described using stochastic differential equations, where random processes such as Gaussian white noise are included in the model. In such formulations, system states become random variables, and control objectives are expressed in terms of expected values or probability distributions.

Stochastic uncertainty modeling is fundamental to stochastic control and estimation theory. Methods such as Linear Quadratic Gaussian (LQG) control and Kalman filtering rely on probabilistic descriptions of uncertainty to compute optimal control actions and state estimates. While stochastic models can provide less conservative and more realistic control strategies, they require accurate statistical information. In practice, probability distributions may be difficult to identify or may change over time. Additionally, stochastic control algorithms often involve significant computational effort, which can limit their applicability in strict real-time environments.

Set-Based and Interval Models

Set-based and interval models represent uncertainty using sets rather than precise numerical values or probability distributions. In this approach, uncertain parameters and states are assumed to belong to known sets, such as intervals, ellipsoids, or polytopes. This representation is particularly useful for structured uncertainty, where dependencies between uncertain parameters must be preserved.

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Interval analysis is one of the simplest set-based techniques, where each uncertain parameter is bounded by lower and upper limits. More advanced models use polytopic uncertainty, where the system is described as a convex combination of multiple vertex systems. Such models are commonly applied in linear parameter-varying (LPV) systems.

Set-based representations offer a balance between deterministic and stochastic approaches. They allow uncertainty to be handled systematically without requiring detailed probabilistic information. In real-time control, set-based methods are attractive because they can be combined with reachability analysis and constraint satisfaction techniques.
However, set-based models may also suffer from conservatism, particularly when uncertainty sets are large. Additionally, the computational complexity of set operations can increase rapidly with system dimension, posing challenges for real-time implementation.

Data-Driven Uncertainty Models

Data-driven uncertainty models have gained attention due to advances in sensing, computation, and machine learning. Instead of relying solely on physical modeling, these approaches use historical and real-time data to characterize uncertainty and system behavior.

In data-driven models, uncertainty is inferred from observed variations in system response. Techniques such as regression, Gaussian process modeling, and neural networks are used to learn uncertain dynamics or disturbance patterns directly from data. These models can adapt to changing conditions and capture complex nonlinear relationships that are difficult to model analytically.

For real-time control, data-driven uncertainty modeling offers flexibility and improved accuracy in dynamic environments. Learning-based models can update uncertainty estimates online, enabling adaptive and predictive control strategies. However, their performance strongly depends on data quality and availability.

A major concern with data-driven approaches is the lack of rigorous stability and robustness guarantees. Since learned models may extrapolate poorly outside the training data, ensuring safe operation under unseen conditions remains challenging. Consequently, hybrid approaches

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that combine data-driven models with classical uncertainty representations are increasingly being explored.

CLASSICAL CONTROL APPROACHES UNDER UNCERTAINTY

Robust Control

Robust control aims to ensure stability and acceptable performance for all admissible uncertainties. The controller is designed to handle worst-case scenarios, often leading to conservative designs. H-infinity control is a widely used robust control technique.

Robust controllers are attractive for real-time applications because they do not require online parameter estimation. However, their conservatism may reduce performance under nominal conditions.

Adaptive Control

Adaptive control addresses uncertainty by adjusting controller parameters online based on observed system behavior. This approach is particularly useful when system parameters are unknown but slowly varying.

In real-time environments, adaptive control offers improved performance but may suffer from stability issues if adaptation is too aggressive or if disturbances are significant.

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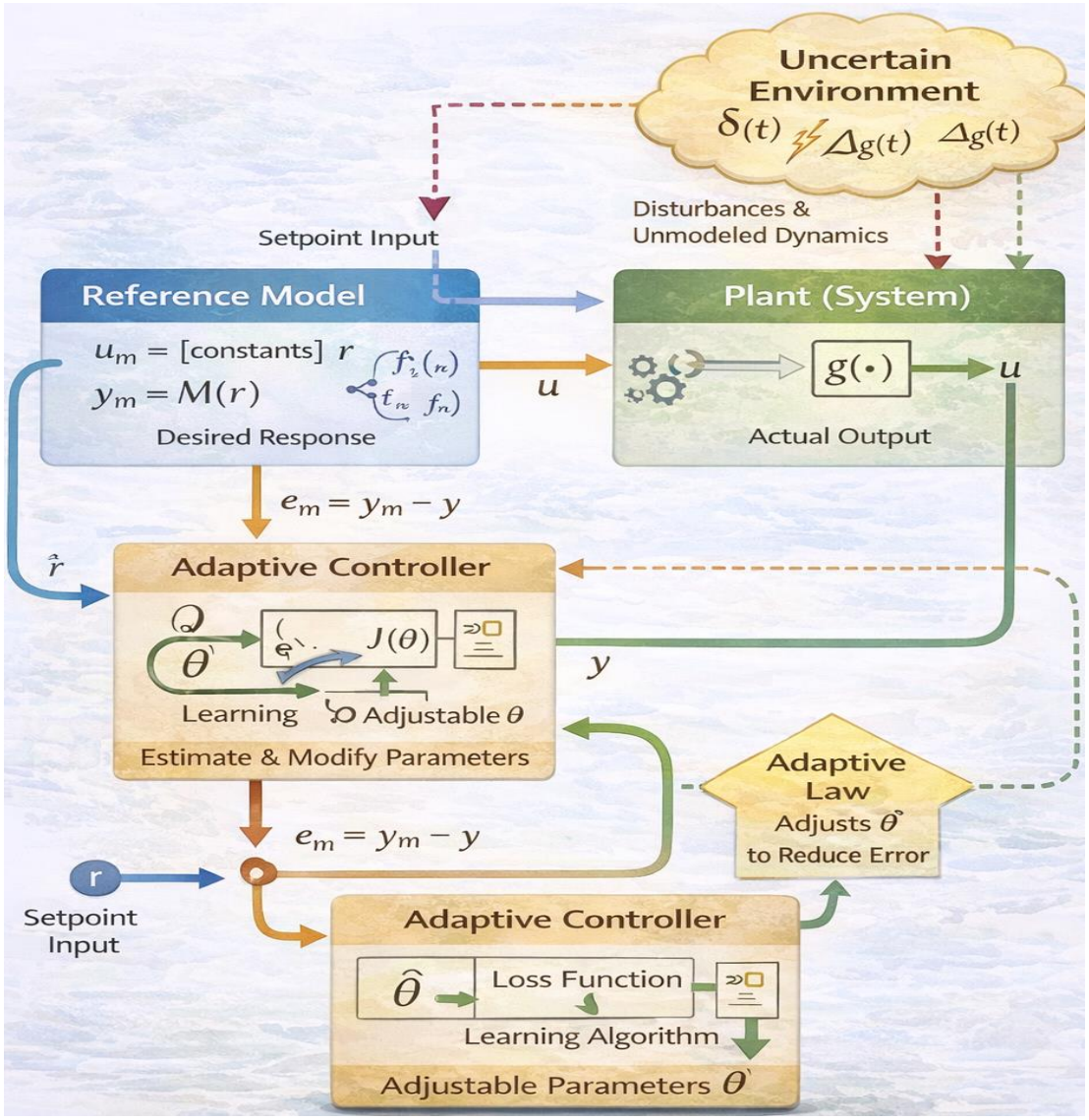


Figure 1: Conceptual Structure of Adaptive Control with Uncertainty

STOCHASTIC CONTROL AND FILTERING

Stochastic control explicitly considers random uncertainties in system dynamics and measurements. The objective is typically to minimize an expected cost function.

Linear Quadratic Gaussian (LQG) Control

LQG control combines linear quadratic regulation with Kalman filtering for state estimation. It is suitable for systems with Gaussian noise and linear dynamics.

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Real-Time Constraints

Although stochastic control provides optimal solutions in a probabilistic sense, its real-time implementation can be challenging due to computational complexity. Approximate solutions are often used in practice.

MODEL PREDICTIVE CONTROL WITH UNCERTAINTY

Model Predictive Control (MPC) has gained significant attention in real-time applications due to its ability to handle constraints and multivariable systems.

Deterministic MPC

Deterministic MPC uses a nominal model and ignores uncertainty during optimization. While simple, this approach may lead to constraint violations under uncertainty.

Robust and Stochastic MPC

Robust MPC accounts for bounded uncertainty, while stochastic MPC incorporates probabilistic constraints. These methods improve reliability but increase computational burden.

Table 2: Comparison of MPC Variants for Real-Time Control

MPC Type	Uncertainty Handling	Computational Cost	Real-Time Suitability
Deterministic MPC	Ignored	Low	High
Robust MPC	Bounded worst-case	Medium–High	Medium
Stochastic MPC	Probabilistic	High	Limited

UNCERTAINTY QUANTIFICATION IN REAL-TIME CONTROL

Uncertainty quantification (UQ) provides tools to measure and propagate uncertainty through control systems. In real-time control, UQ must be computationally efficient.

Methods such as Monte Carlo simulation are often too slow for real-time use. Reduced-order models, polynomial chaos expansions, and online learning techniques are increasingly used to estimate uncertainty with limited computation.

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DATA-DRIVEN AND LEARNING-BASED APPROACHES

Recent advances in machine learning have influenced real-time control under uncertainty. Reinforcement learning and neural network-based controllers can learn control policies directly from data.

While these methods show promise, they raise concerns regarding stability, interpretability, and safety. Hybrid approaches that combine learning with classical control theory are an active area of research.

PRACTICAL CHALLENGES AND APPLICATIONS

Computational Constraints

Real-time controllers must operate within strict time limits. Algorithms with high computational complexity may not be feasible, even if they offer superior performance.

Safety and Reliability

In safety-critical applications, such as aerospace and medical devices, controllers must guarantee stability despite uncertainty. Certification and validation remain major challenges.

Application Domains

Real-time control with uncertainty is applied in autonomous vehicles, robotic surgery, renewable energy systems, and industrial automation. Each domain presents unique uncertainty characteristics and performance requirements.

FUTURE RESEARCH DIRECTIONS

Future research is expected to focus on integrating uncertainty quantification with learning-based control, developing scalable algorithms for large systems, and improving theoretical guarantees for real-time performance. There is also a growing need for benchmark problems and experimental validation.

CONCLUSION

Real-time control with uncertainty is a critical and challenging area in modern control theory. Uncertainty arises from multiple sources and cannot be ignored in practical systems. This paper reviewed key concepts, mathematical models, and control strategies used to address uncertainty

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in real-time environments. Classical robust and adaptive control methods remain relevant, while stochastic and predictive control offer advanced capabilities at higher computational cost. Emerging data-driven approaches provide new opportunities but also introduce new risks. Overall, effective real-time control requires a careful balance between performance, robustness, and computational efficiency. Continued research in this area is essential for the safe and reliable operation of future engineering systems.

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