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Types of Fuzzy Implicative Filters of Lattice Pseudo Wajsberg Algebras

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Abstract

In this paper, we introduce the notion of Type-1fuzzy implicative filter of lattice pseudo-Wajsberg algebra and we obtain some related properties. Further, we introduce Type-2 fuzzy implicative filter of lattice pseudo-Wajsberg algebra, we show that characterization of Type-1and Type-2fuzzyimplicative filters of lattice pseudo Wajsberg algebra.

Keywords: *Wajsberg algebra; Pseudo-Wajsberg algebra; Implicative filter; Fuzzy implicative filter; Type-1, Type-2fuzzy implicative filters*

Mathematical Subject classification 2010: *03E70, 03E72, 03G10.*

INTRODUCTION

The concept of a fuzzy set (fuzzy subset) introduced by Zadeh [10] in 1965, provides a natural generalization for treating mathematically the fuzzy phenomena which exist pervasively in our real world and for building new branches of fuzzy Mathematics. In fuzzy set theory, the member of an element to a fuzzy set is a single value between 0 and 1. MordchajWajsbreg introduced the concept of Wajsberg algebras in 1935 and studied by Font, Rodriguez and Torrenns in [4]. They [4] defined lattice structure of Wajsberg algebras. Also, they [4] introduced the notion of an implicative filter of lattice Wajsberg algebras and discussed some of their properties. Basheer Ahamed and Ibrahim [1] introduced the definition of fuzzy implicativefilters of lattice Wajsberg algebra and obtained some properties

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with illustrations. Pseudo-Wajsberg algebras are generalizations of Wajsberg algebras. Pseudo-Wajsberg algebras were introduced by RodicaCeterchi [7] with the explicit purpose of providing a concept categorically equivalent to that of pseudo-MV algebras. The authors [5, 6] introduced the notions of implicative filter and fuzzy implicative filter of lattice pseudo-Wajsbergalgebra and investigated some of their properties.

The aim of this paper is to introduce the notion of Type-1 fuzzy implicative filter of lattice pseudo-Wajsberg algebras, and investigate some properties with illustrations. Further, we investigate Type-2 fuzzy implicative filter of lattice pseudo-Wajsbergalgebras, and we discuss some properties.

PRELIMINARIES

In this section, we recall some basic definitions and their properties which are helpful to develop the main results.

Definition 2.1[1] An algebra $(A, \rightarrow, ^{-}, 1)$ with a binary operation $" \rightarrow "$ and a quasi complement $" ^{-} "$ is called a Wajsberg algebra if it satisfies the following axioms for all $x, y, z \in A$,

(i) $1 \rightarrow x = x$

(ii) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$

(iii) $(x \rightarrow y) \rightarrow \big((y \rightarrow z) \rightarrow (x \rightarrow z)\big) = 1$

(iv) $(x^{-} \rightarrow y^{-}) \rightarrow (y \rightarrow x) = 1.$

Definition 2.2[1]A Wajsberg algebra A is called a lattice Wajsberg algebra if it satisfies the following axioms for all $x, y \in A$,

(i) The partial ordering $" \leq "$ on a lattice Wajsberg algebra A, such that $x \leq y$ if and only if $x \rightarrow y = 1$

(ii) $(x \vee y) = (x \rightarrow y) \rightarrow y$

(iii) $(x \wedge y) = \big((x^{-} \rightarrow y^{-}) \rightarrow y^{-}\big)$.

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Definition 2.3[1] Let A be Wajsberg algebra. A non-empty subset F of A is called an implicative filter of A if it satisfies the following axioms for all $x, y \in A$,

(i) $1 \in F$

(ii) $x \in F$ and $x \to y \in F$ imply $y \in F$.

Definition 2.4[7]. An algebra $(A, \to, \rightsquigarrow, ^{-}, ^{\sim}, 1)$ with a binary operations " $\to$ "," $\rightsquigarrow$ " and quasi complements " $^{-}$", " $^{\sim}$ " is called a pseudo-Wajsberg algebra if it satisfies the following axioms for all $x, y, z \in A$,

(i) (a) $1 \to x = x$

(b) $1 \rightsquigarrow x = x$

(ii) $(x \rightsquigarrow y) \to y = (y \rightsquigarrow x) \to x = (y \to x) \rightsquigarrow x = (x \to y) \rightsquigarrow y$

(iii)(a) $(x \to y) \to \big((y \to z) \rightsquigarrow (x \to z)\big) = 1$

(b) $(x \rightsquigarrow y) \rightsquigarrow \big((y \rightsquigarrow z) \to (x \rightsquigarrow z)\big) = 1$

(iv) $1^{-} = 1^{\sim} = 0$

(v) (a) $(x^{-} \rightsquigarrow y^{-}) \to (y \to x) = 1$

(b) $(x^{\sim} \to y^{\sim}) \rightsquigarrow (y \rightsquigarrow x) = 1$

(vi) $(x \to y^{-})^{\sim} = (y \rightsquigarrow x^{\sim})^{-}$

Definition 2.5[7] Algebra $(A, \to, \rightsquigarrow, ^{-}, ^{\sim}, 1)$ is called a lattice pseudo-Wajsberg algebras if it satisfies the following axioms for all $x, y \in A$,

(i) A partial ordering "$\leq$" on a lattice pseudo-Wajsberg algebra A, such that x $\leq$ y if and only if $x \to y = 1 \Leftrightarrow x \rightsquigarrow y = 1$.

(ii) $x \vee y = (x \to y) \rightsquigarrow y = (y \to x) \rightsquigarrow x = (x \rightsquigarrow y) \to y = (y \rightsquigarrow x) \to x$

(iii) $x \wedge y = (x \rightsquigarrow (x \to y)^{\sim})^{-} = \big((x \to y) \to x^{-}\big)^{\sim}$

$= (y \rightsquigarrow (y \to x)^{\sim})^{-} = \big((y \to x) \to y^{-}\big)^{\sim}$

$= (y \to (y \rightsquigarrow x)^{-})^{\sim} = \big((y \rightsquigarrow x) \rightsquigarrow y^{\sim}\big)$

$= (x \to (x \rightsquigarrow y)^{-})^{\sim} = \big((x \rightsquigarrow y) \rightsquigarrow x^{\sim}\big)$

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Proposition 2.6[1] A Wajsberg algebra $(A, \to, \bar{}, 1)$ satisfies the following axioms for all $x, y \in A$,

(i) $x \to x = 1$

(ii) If $x \to y = y \to x = 1$, then $x = y$

(iii) $x \to 1 = 1$

(iv) $x \to (y \to x) = 1$

(v) If $x \to y = y \to z = 1$, then $x \to z = 1$

(vi) $(x \to y) \to \big((z \to x) \to (z \to y)\big) = 1$

(vii) $x \to (y \to z) = y \to (x \to z)$.

Proposition 2.7[7] A lattice pseudo-Wajsberg algebra $(A, \to, \rightsquigarrow, \bar{}, \tilde{}, 1)$ satisfies the following axioms for all $x, y \in A$,

(i) $x \to x = 1, x \rightsquigarrow x = 1$

(ii) $x \to (y \rightsquigarrow x) = 1,\ x \rightsquigarrow (y \to x) = 1$

(iii) $x \leq y \Rightarrow z \to x \leq z \to y;\ z \rightsquigarrow x \leq z \rightsquigarrow y$

(iv) $x \leq y \Rightarrow y \to z \leq x \to z;\ y \rightsquigarrow z \leq x \rightsquigarrow z$

(v) $x \leq y \to x;\ x \leq y \rightsquigarrow x$

(vi) $x \to y \leq (y \to z) \rightsquigarrow (x \to z);\ x \rightsquigarrow y \leq (y \rightsquigarrow z) \to (x \rightsquigarrow z)$

(vii) $x \to y \leq (z \to x) \to (z \to y);\ x \rightsquigarrow y \leq (z \rightsquigarrow x) \rightsquigarrow (z \rightsquigarrow y)$

(viii) $x \to (y \rightsquigarrow z) = y \rightsquigarrow (x \to z)$.

Definition 2.8[5] Let A be lattice pseudo-Wajsberg algebra. A non-empty subset F of A is called an implicative filter of A if it satisfies the following axioms for all $x, y \in A$,

(i) $1 \in F$

(ii) $x \in F$ and $x \to y \in F$ imply $y \in F$

(iii) $x \in F$ and $x \rightsquigarrow y \in F$ imply $y \in F$.

Proposition 2.9[5] Let F be any implicative filter of lattice pseudo-Wajsberg algebra of A satisfying the axiom $x \rightsquigarrow (y \rightsquigarrow z) = y \rightsquigarrow (x \rightsquigarrow z)$ for all $x, y \in A$.

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Definition 2.10[1] Let A be a set. A function $\mu : A \to [0,1]$ is called a fuzzy subset on A, for each $x \in A$, the value of $\mu(x)$ describes a degree of membership of x in μ.

Definition 2.11[1] Let A be lattice Wajsberg algebra. A non-empty fuzzy subset μ of A is called

(i) $\mu(1) \geq \mu(x)$

(ii) $\mu(y) \geq \min\{\mu(x \to y), \mu(x)\}$

Definition 2.12 [6] Let A be lattice pseudo-Wajsbergalgebra. A non-empty fuzzy subset μ of A is called an implicative filter of A if it satisfies the following axioms for all $x, y \in A$,

(i) $\mu(1) \geq \mu(x)$

(ii) $\mu(y) \geq \min\{\mu(x \to y), \mu(x)\}$

(iii) $\mu(y) \geq \min\{\mu(x \rightsquigarrow y), \mu(x)\}$.

MAIN RESULTS

3.1. Type -1 fuzzy implicative filter of lattice pseudo -Wajsberg algebra

In this section, we introduce Type-1fuzzy implicative filter of lattice pseudo-Wajsberg algebra, and investigate some properties with illustrations.

Definition 3.1.1 Let A be lattice pseudo-Wajsberg algebra. A non-empty fuzzy subset μ of A is called aType-1 implicative filter of A if it satisfies the following axioms for all $x, y \in A$,

(i) $\mu(1) \geq \mu(x)$

(ii) $\mu(y) \geq \min\{\mu(x \to y), \mu(x)\}$

(iii) $\mu(y) \geq \min\{\mu(x \rightsquigarrow y), \mu(x)\}$

(iv) $\mu(x) \geq \mu((x \to y) \to x)$; $\mu(x) \geq \mu((x \rightsquigarrow y) \rightsquigarrow x)$

Example 3.1.2

Consider a set A={0,a,b,c,1}.

Define a partial ordering "≤" on A, such that 0<a<b,c<1 and the binary operations "→" ,"⇝" and quasi complements " ¬" , " ˜ " given by the following tables (1), (2), (3) and (4).

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Table: 1

x	x^-
0	1
a	c
b	c
c	0
1	0

Table: 2

$\rightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	c	c	1	1	1
c	0	b	b	1	1
1	0	a	b	c	1

Table: 3

x	$x^{\sim}$
0	1
a	b
b	0
c	b
1	0

Table: 4

$\rightsquigarrow$	0	a	b	c	1
0	1	1	1	1	1
a	b	1	1	1	1
b	0	c	1	1	1
c	b	b	b	1	1
1	0	a	b	c	1

Consider the fuzzy subset μ on$A$as, $\mu(x) = \begin{cases} 0.7 & if \ \ x = 1 \\ 0.3 & Otherwise \end{cases}$ for all$x \in A$. Then, we have μ is Type-1 fuzzy implicative filter of lattice pseudo-Wajsbergalgebra A.

Example 3.1.3. Consider a set $A = \{0, a, b, c, d, 1\}$. Define a partial ordering "$\leq$" on A, such that
$0 < a, b < c < d < 1$and the binary operations $" \rightarrow ", " \rightsquigarrow "$ and quasi complements $" \ ^- ", " \ ^{\sim} "$ given by the following tables (5), (6), (7) and (8).

Table: 5

x	x^-
0	1
a	d
b	d
c	d
d	0
1	0

Table: 6

$\rightarrow$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	d	1	d	1	1	1
b	d	d	1	1	1	1
c	d	d	d	1	1	1
d	0	a	b	c	1	1
1	0	a	b	c	d	1

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Table: 7

x	$x^{\sim}$
0	1
a	c
b	c
c	c
d	c
1	0

Table: 8

$\rightsquigarrow$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	c	1	c	1	1	1
b	c	c	1	1	1	1
c	c	c	c	1	1	1
d	c	c	c	c	1	1
1	0	a	b	c	d	1

Consider the fuzzy subset μ on$A$as, $\mu(x) = \begin{cases} 0.8 & if \ \ x \in \{1, a, b, c\} \\ 0.6 & if \ \ x \in \{0, d\} \end{cases}$ for all $x \in A$.

Then, we have μ is nota Type-1 fuzzy implicative filter of lattice pseudo-Wajsbergalgebra A.

Since, we have $\mu(d) = 0.6$, but $\mu((d \to 0) \to d) = 0.8$

Thus, we have $\mu(d) \not\geq \mu((d \to 0) \to d) = 0.6 \not\geq 0.8$.

Remark 3.1.4. Let μ be a Type-1 fuzzy implicative filter of lattice pseudo-Wajsbergalgebra, then for all $x, y \in A$, $x \leq y$ implies $\mu(x) \leq \mu(y)$.

Proposition 3.1.5. A fuzzy subset μ of A is aType-1implicative filter of lattice pseudo-Wajsbergalgebra A if and only if for all $x, y, z \in A$, $x \leq y \to z$ and$x \leq y \rightsquigarrow z$ implies $\mu(z) \geq \min\{\mu(x), \mu(y)\}$.

Proof. Let μ is fuzzy subset of A and satisfying the conditions$x \leq y \to z$ and$x \leq y \rightsquigarrow z$ implies $\mu(z) \geq \min\{\mu(x), \mu(y)\}$.

$x \leq x \to 1$ and $x \leq x \rightsquigarrow 1$ for all $x \in A$,
$\mu(1) \geq \min\{\mu(x), \mu(x)\} = \mu(x)$.

Since,

Hence, we have $\mu(1) \geq \mu(x)$.

Since $x \to y \leq x \to y$ and $x \rightsquigarrow y \leq x \rightsquigarrow y$

Hence, we have$\mu(y) \geq \min\{\mu(x \to y), \mu(x)\}$and $\mu(y) \geq \min\{\mu(x \rightsquigarrow y), \mu(x)\}$

Consider $x \geq (x \to y) \to x$ and μ is afuzzy implicative filter,

We have $\mu(x) \geq \mu((x \to y) \to x)$ [Fromremark 3.1.4]

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Similarly, we show $\mu(x) \geq \mu((x \rightsquigarrow y) \rightsquigarrow x)$

Therefore, μ is a Type-1 fuzzy implicative filter of lattice pseudo-Wajsbergalgebra.

Conversely, if fuzzy subset μ is a Type-1 implicative filter of lattice pseudo-Wajsbergalgebra A.

Let $x, y, z \in A$, if $x \leq y \rightarrow z$ and $x \leq y \rightsquigarrow z$, then $\mu(x) \leq \mu(y \rightarrow z)$ and $\mu(x) \leq \mu(y \rightsquigarrow z)$

[Fromremark 3.1.4]

We have

$\mu(z) \geq \min\{\mu(y \rightarrow z), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}$

And

$\mu(z) \geq \min\{\mu(y \rightsquigarrow z), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}$.

Hence, for all $x, y, z \in A$, $x \leq y \rightarrow z$ and $x \leq y \rightsquigarrow z$ implies $\mu(z) \geq \min\{\mu(x), \mu(y)\}$.

Proposition 3.1.6. A fuzzy subset μ of A is called a Type-1 implicative filter of lattice pseudo-Wajsbergalgebra A if and only if (i) $\mu(x) \geq \mu((x \rightarrow y) \rightarrow x)$ and $\mu(x) \geq \mu((x \rightsquigarrow y) \rightsquigarrow x)$ for all $x, y \in A$.

Proof. **(i)** Let μ is a Type-1 implicative filter of lattice pseudo-Wajsbergalgebra A. Let $x, y \in A$ and $x \geq (x \rightarrow y) \rightarrow x$, Since μ is aType-1 fuzzy implicative filter,

$\mu(x) \geq \mu((x \rightarrow y) \rightarrow x)$

[Fromremark 3.1.4]

Similarly, $x \geq (x \rightsquigarrow y) \rightsquigarrow x$ and μ is a Type-1fuzzy implicative filter,

we have $\mu(x) \geq \mu((x \rightsquigarrow y) \rightsquigarrow x)$.

[Fromremark 3.1.4]

Therefore, $\mu(x) \geq \mu((x \rightarrow y) \rightarrow x)$; $\mu(x) \geq \mu((x \rightsquigarrow y) \rightsquigarrow x)$

For all $x, y \in A$.

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Conversely, to proveμ is a Type-1 implicative filter of lattice pseudo-Wajsbergalgebra A

Clearly, $\mu(1) \geq \mu(x)$ for all $x \in A$

We have, $\mu(y) = \mu((y \to x) \to y) \geq \min\{\mu((x \to y) \to x), \mu(x)\}$

$\mu(y) \geq \min\{\mu(x \to y), \mu(x)\}$ for all $x, y \in A$.

We have,

$\mu(y) = \mu((y \rightsquigarrow x) \rightsquigarrow y) \geq \min\{\mu((x \rightsquigarrow y) \rightsquigarrow x), \mu(x)\}$

$\mu(y) \geq \min\{\mu(x \rightsquigarrow y), \mu(x)\}$ for all $x, y \in A$.

Obviously, $\mu(x) \geq \mu((x \to y) \to x)$; $\mu(x) \geq \mu((x \rightsquigarrow y) \rightsquigarrow x)$

For all $x, y \in A$.

Hence, μ is a Type-1 fuzzy implicative filter of lattice pseudo-Wajsbergalgebra A

3.2. Type -2 fuzzy implicative filter of lattice pseudo -Wajsberg algebra

In this section, we introduce Type-2 fuzzy implicative filter of lattice pseudo-Wajsberg algebra, and investigate some properties with illustrations.

Definition 3.2.1. Let A be lattice pseudo-Wajsberg algebra. A non-empty fuzzy subset μ of A is called aType-2implicative filter of A if it satisfies the following axioms for all $x, y \in A$,

(i) $\mu(1) \geq \mu(x)$
(ii) $\mu(y) \geq \min\{\mu(x \to y), \mu(x)\}$
(iii) $\mu(y) \geq \min\{\mu(x \rightsquigarrow y), \mu(x)\}$
(iv) $\mu(x) \geq \mu((x \to y) \rightsquigarrow x)$; $\mu(x) \geq \mu((x \rightsquigarrow y) \to x)$

Example 3.2.2 Consider a set A={0,a,b,c,1}. Define a partial ordering "≤" on A, such that 0<a<b,c<1 and the binary operations "→" ,"⇝" and quasi complements " ¬" , " ˜ " given by the following tables (1), (2), (3) and (4).

Consider the fuzzy subset μ on$A$as, $\mu(x) = \begin{cases} 0.2 & if\ x = 1 \\ 0.8 & if\ x \in \{0, a, b, c\} \end{cases}$forall$x \in A$.Then, we have μ is Type-2 fuzzy implicative filter of lattice pseudo-Wajsberg algebra A.

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Table: 9

x	x^-
0	1
a	c
b	b
c	b
1	0

Table: 10

$\rightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	c	1	1	1
c	b	c	c	1	1
1	0	a	b	c	1

Table: 11

x	$x^{\sim}$
0	1
a	c
b	c
c	a
1	0

Table: 12

$\rightsquigarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	c	c	1	1	1
c	a	c	c	1	1
1	0	a	b	c	1

Example 3.2.3 Consider a set $A = \{0, a, b, c, d, 1\}$. Define a partial ordering "$\leq$" on A, such that $0 < a, b < c < d < 1$ and the binary operations $" \rightarrow ", " \rightsquigarrow "$ and quasi complements $" \ ^- ", " \ ^{\sim} "$ given by the following tables (5), (6), (7) and (8).

Table: 13

x	x^-
0	1
a	d
b	d
c	d
d	0
1	0

Table:14

$\rightarrow$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	d	1	d	1	1	1
b	d	d	1	1	1	1
c	d	d	d	1	1	1
d	0	a	b	c	1	1
1	0	a	b	c	d	1

Table:15

x	$x^{\sim}$
0	1
a	c
b	c
c	c
d	c
1	0

Table:16

$\rightsquigarrow$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	c	1	c	1	1	1
b	c	c	1	1	1	1
c	c	c	c	1	1	1
d	c	c	c	c	1	1
1	0	a	b	c	d	1

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Consider the fuzzy subset μ on$A$as, $\mu(x) = \begin{cases} 0.7 & if \ \ x \in \{1, a, b\} \\ 0.5 & if \ \ x \in \{0, c, d\} \end{cases}$ for all $x \in A$.

Then, we have μ is nota Type-2 fuzzy implicative filter of lattice pseudo-Wajsbergalgebra A.

Since,Since, we have $\mu(d) = 0.5$, but $\mu((d \to 0) \rightsquigarrow d) = 0.7$

Thus, we have $\mu(d) \not\geq \mu((d \to 0) \rightsquigarrow d) = 0.5 \not\geq 0.7$.

Remark 3.2.4 Let μ be a Type-2 fuzzy implicative filter of lattice pseudo-Wajsbergalgebra, then for all $x, y \in A$, $x \leq y$ implies $\mu(x) \leq \mu(y)$.

Proposition 3.2.5 A fuzzy subset μ of A is aType-2implicative filter of lattice pseudo-Wajsbergalgebra A if and only if for all $x, y, z \in A$, $x \leq y \to z$ and$x \leq y \rightsquigarrow z$ implies $\mu(z) \geq \min\{\mu(x), \mu(y)\}$.

***Proof.*(i)** Let μ is fuzzy subset of A and satisfying the condition$x \leq y \to z$ and$x \leq y \rightsquigarrow z$ implies $\mu(z) \geq \min\{\mu(x), \mu(y)\}$.

Since,

$x \leq x \to 1$ and $x \leq x \rightsquigarrow 1$ for all $x \in A$,
$\mu(1) \geq \min\{\mu(x), \mu(x)\} = \mu(x)$.

Hence, we have $\mu(1) \geq \mu(x)$.

Since $x \to y \leq x \to y$ and $x \rightsquigarrow y \leq x \rightsquigarrow y$

Hence, we have$\mu(y) \geq \min\{\mu(y \to x), \mu(x)\}and\mu(y) \geq \min\{\mu(y \rightsquigarrow x), \mu(x)\}$

Consider $x \geq (x \to y) \rightsquigarrow x$ and μ is afuzzy implicative filter,

We have $\mu(x) \geq \mu((x \to y) \rightsquigarrow x)$

[Fromremark 3.2.4]

Similarly, we show$\mu(x) \geq \mu((x \rightsquigarrow y) \to x)$

Therefore, μ is a Type-2fuzzy implicative filter of lattice pseudo-Wajsberg algebra.

Conversely, if fuzzy subset μ is a Type-2implicative filter of lattice pseudo-Wajsberg algebra A.

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Let

$x, y, z \in A$, if $x \leq y \to z$ and $x \leq y \rightsquigarrow z$, then $x \leq y \to z$ and $x \leq y \rightsquigarrow z$.

Then $\mu(x) \leq \mu(y \to z)$ and $\mu(x) \leq \mu(y \rightsquigarrow z)$

[Fromremark 3.2.4]

We have

$\mu(z) \geq \min\{\mu(y \to z), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}$

and

$\mu(z) \geq \min\{\mu(y \rightsquigarrow z), \mu(y)\} \geq \min\{\mu(x), \mu(y)\}$.

Hence, for all

$x, y, z \in A$, $x \leq y \to z$ and $x \leq y \rightsquigarrow z$ implies $\mu(z) \geq \min\{\mu(x), \mu(y)\}$.

Proposition 3.2.6 A fuzzy subset μ of A is called a Type-2 implicative filter of lattice pseudo-Wajsberg algebra A if and only if

$\mu(x) \geq \mu((x \to y) \rightsquigarrow x)$; $\mu(x) \geq \mu((x \rightsquigarrow y) \to x)$

for all $x, y \in A$.

Proof. **(i)** Let μ is a Type-2 implicative filter of lattice pseudo-Wajsbergalgebra A. Let $x, y \in A$ and $x \geq (x \to y) \rightsquigarrow x$, Since μ is aType-2 fuzzy implicative filter,

we have, $\mu(x) \geq \mu((x \to y) \rightsquigarrow x)$ [Fromremark 3.2.4]

Similarly, $x \geq (x \rightsquigarrow y) \to x$ and μ is aType-2 fuzzy implicative filter,

we have, $\mu(x) \geq \mu((x \rightsquigarrow y) \to x)$.

[Fromremark 3.2.4]

Therefore,

$\mu(x) \geq \mu((x \to y) \rightsquigarrow x)$; $\mu(x) \geq \mu((x \rightsquigarrow y) \to x)$

for all $x, y \in A$.

Conversely, to prove μ is a Type-2 implicative filter of lattice pseudo-Wajsbergalgebra A

Clearly, $\mu(1) \geq \mu(x)$ for all $x \in A$.

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We have, $\mu(y) = \mu((y \to x) \to y) \geq \min\{\mu((x \to y) \to x), \mu(x)\}$

$\mu(y) \geq \min\{\mu(x \to y), \mu(x)\}$ for all $x, y \in A$.

We have, $\mu(y) = \mu((y \rightsquigarrow x) \rightsquigarrow y) \geq \min\{\mu((x \rightsquigarrow y) \rightsquigarrow x), \mu(x)\}$

$\mu(y) \geq \min\{\mu(x \rightsquigarrow y), \mu(x)\}$ for all $x, y \in A$.

Obviously,

$\mu(x) \geq \mu((x \to y) \rightsquigarrow x)$; $\mu(x) \geq \mu((x \rightsquigarrow y) \to x)$

for all $x, y \in A$.

Hence, μ is a Type-2 fuzzy implicative filter of lattice pseudo-Wajsbergalgebra A.

Proposition 3.2.7. Let μ is a Type-1 and Type-2 fuzzy implicative filter of lattice pseudo-Wajsberg algebra of A satisfying the following axioms for all $x, y \in A$,

(i) $\mu((x \to 1) \to (y \to 1)) = \mu((x \to y) \to 1)$

(ii) $\mu((x \rightsquigarrow 1) \rightsquigarrow (y \rightsquigarrow 1)) = \mu((x \rightsquigarrow y) \rightsquigarrow 1)$

(iii) $\mu((x \to 1) \rightsquigarrow (y \to 1)) = \mu((x \to y) \to 1)$

(iv) $\mu((x \rightsquigarrow 1) \to (y \rightsquigarrow 1)) = \mu((x \rightsquigarrow y) \rightsquigarrow 1)$

***Proof.*(i)** Letμ is a Type-1 and Type-2 implicative filter of lattice pseudo-Wajsbergalgebra A.

Let x, y

$\in A$,

$\mu((x \to 1) \to (y \to 1)) = \min\{\mu(1), \mu((x \to 1) \to (y \to 1))\} \geq \mu((x \to y) \to 1)$

We have $(x \to y) \to 1 \leq (x \to 1) \to (y \to 1)$

$\mu((x \to 1) \to (y \to 1)) \leq \mu((x \to y) \to 1)$

[Fromremark 3.1.4]

Therefore, $\mu((x \to 1) \to (y \to 1)) = \mu((x \to y) \to 1)$.

Similarly, we show (ii),(iii),(iv).

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CONCLUSION

In this paper, we have introduced the notion of Type-1 and Type-2 fuzzy implicative filters of lattice pseudo-Wajsberg algebra. The properties and equivalent conditions of Type-1and Type-2 fuzzy implicative filters of lattice pseudo-Wajsberg algebra are discussed. Further, we extend this idea as intuitionistic fuzzy of the implicative filter of lattice pseudo-Wajsberg algebra.

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