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New Characterization of $\hat{\mu}\beta$ Kernel Set in Fuzzy Topological Spaces

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Abstract

In this paper we introduce a kernelled fuzzy point, boundary kernelled fuzzy point and derived kernelled fuzzy point of a subset A of X, and using these notions to define kernel set of fuzzytopological spaces. Also, we introduce fuzzy topological kr space. The investigation enables us topresent some new fuzzy separation axioms between FT_0 and FT_1spaces.

Keywords: *Fuzzy Topological Space, Kernelled Fuzzy Point, Boundary Kernelled Fuzzy Point, Derived Kernelled Fuzzy Point, Kernel Set, Weak Fuzzy Separation Axioms,FR_i space, i = 0,1*

INTRODUCTION

The concept of fuzzy set and fuzzy set operations were first introduced by L. A. Zadeh in 1965 [7]. After Zadeh 's introduction of fuzzy sets, Chang [2] defined and studied the notion of fuzzytopological space in 1968. In 1997, fuzzy generalized closed set (Fg - closed set) was introduced byG. Balasubramania and P. Sundaram [5]. In 1998, the notion of Fgs - closed set was defined and investigated by H. Maki el al [6]. In 2002, O. Bedre Ozbakir [9], defined the concept of fuzzy generalized strongly closed set. In 1984, fuzzy separation axioms have been introduced and investigated by A. S. Mashour and others [1].In this paper we introduce a new characterization of $\hat{\mu}\beta$kernel set through our definition($\hat{\mu}$)βkernelled fuzzy point, boundary $\hat{\mu}\beta$kernelled fuzzy point and derived $\hat{\mu}\beta$kernelled fuzzy point. By these notions, we obtain that the kernel of a set in fuzzy topological space(X,τ) is a union of the set itself with the set

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of all boundary μ Ɓ kernelled fuzzy points. In addition, it is a union of the set itself with the set of all derived μ Ɓ kernelled fuzzy points and we give some result of FR_0- space by using these notions. Also in this paper we introduce fuzzy topological kr- space iff kernel of a subset A of X is an fuzzy open set. Via this kind of fuzzy topological space, we give a new characterization of fuzzy separation axioms lying between FT_0and FT_1spaces.

PRELIMINARIES

Fuzzy sets theory, introduced by Lotfi. A. Zada in 1965 [7], is the extension of classical settheory by allowing the membership of elements to range from 0 to 1. Let *X* be the universe of a classical set of objects. Membership in a classical subset *A* of *X* is often viewed as a characteristicfunction μ_A from *X* into {0, 1}, where

$$\mu_A(X) = \begin{cases} 1, & \text{for } x \in A \\ 0, & for\ x \notin A \end{cases}$$

for any x ∈ *X*.{0,1} is called a valuation set (see [11]). If the valuation set is allowed to be the real interval [0,1], *A*is called a fuzzy set in *X*. $\mu_A(X)$ (or simply *A*(X)) is the membership value (or degree of membership) of X in *A*. Clearly, *A* is a subset of *X* that hasno sharp boundary. A fuzzy set *A* in *X* can be represented by the set of pairs: *A* ={(x, *A*(x)), x ∈ *X*}.

Let *A* : *X* ⟶[0,1] be a fuzzy set. If *A*(x) = 1, for each x ∈ *X*, we denote it by $1_X$and if *A*(x) = 0, for each x ∈*X*, we denote it by 0_X. That is, by $0_X$and 1_X, we mean the constant fuzzy sets taking the values 0 and 1 on *X*, respectively . Let I =[0,1]. The set of all fuzzy sets in *X*, denoted by I^X[10].

Definition 2.1:[3] Let *A* be a fuzzy set of a set *X*. The support of *A* is the elements x whose membership value is greater than 0, i.e., supp(*A*) = {x ∈ *X* : *A*(x)> 0}.

Definition 2.2:[4] Let *A* and *B* be any two fuzzy sets in *X*. Then we define *A* ⋁*B* : *X*⟶ [0,1] as follows: (*A* ⋁*B*)(x) = *max* {*A*(x), *B*(x)}.Also, we define *A* ∧*B* : *X* ⟶ [0,1] as follows: (*A*∧*B*)(x) = *min*{*A*(x), *B*(x)}.By *A* ⋁*B* (*A* ∧ *B*), we mean the union (intersection) between two fuzzy sets *A* and *B* of *X*.

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Definition 2.3:[3] Let A be any fuzzy set in a set X. The complement of A, is denoted by $1_X - A$ or A^c and defined as follows: $A^c(x) = 1 - A(x)$, for each $x \in X$.

Remark 2.4: From definition (2.2) and definition (2.3), we have, if $A, B \in I^X$, then $A \vee B$, $A \wedge B$ and $1_X - A \in I^X$.

Definition 2.5:[8] A fuzzy point x_λ in a set X is a fuzzy set defined as follows:

$$x_\lambda(y) = \begin{cases} \lambda, & \text{if } y = x \\ 0, & otherwise \end{cases}$$

Where $0 < \lambda \leq 1$. Now, $\text{supp}(x_\lambda) = \{y : x_\lambda(y) > 0\}$, but

$$x_\lambda(y) = \begin{cases} \lambda & \text{if } y = x \\ 0 & otherwise, \end{cases}$$ Where $0 < \lambda \leq 1$.

Then $\text{supp}(x_\lambda) = x$, so the value at x is λ, and call the point x its support of fuzzy point x_λ and λ is the height of x_λ. That is, x_λ has the membership degree 0 for all $y \in X$ except one, say $x \in X$.

Definition 2.6:[2] A fuzzy topology on a set X is a family T of fuzzy sets in X which satisfies the
following conditions:

(i) $0_X, 1_X \in T$,

(ii) If A , $B \in T$, then $A \wedge B \in T$,

(iii) If $\{A_i : i \in J\}$ is a family in $T_{i \in J}$, then $\vee A_i \in T$.

T is called a fuzzy topology for X and the pair (X, T)(or simply X) is a fuzzy topological space or fts for short. Every element of T is called T - fuzzy open set (fuzzy open set, for short). A fuzzy set is T- fuzzy closed (or simply fuzzy closed), if its complement is fuzzy open set. As ordinary topologies,the indiscrete fuzzy topology on X contains only 0_X and 1_X(i.e., $\emptyset$, X) , while the discrete fuzzy topology on X contains all fuzzy sets in X.

Example 2.7: Let $X = [-1,1]$, and let B_1 , B_2 and B_3 are fuzzy sets in X defined as follows:

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$$B_1(x) = \begin{cases} 1, & if -1 \leq x < 0 \\ 0, & if\ 0 \leq x \leq 1 \end{cases}$$

$$B_2(x) = \begin{cases} 0, & if -1 \leq x < 0 \\ 1, & if\ 0 \leq x \leq 1 \end{cases}$$

$$B_3(x) = \begin{cases} 0. & if -1 \leq x < 0 \\ 1/5, & if\ 0 \leq x \leq 1 \end{cases}$$

Let $T = \{ 0_X, B_1, B_2, B_3, B_1 \vee B_3, 1_X \}$, then T is a fuzzy topology on X, and (X, T) is a fts.

Example 2.8: Let $X = [0,1]$ and $T = \{0_X, K, 1_X\}$. Then T is a fuzzy topology on X, where $K : X \longrightarrow [0,1]$ defined as: $K(x) = x^2$, for all $x \in X$, and (X, T) is a fts.

Definition 2.9:[2] Let A be any fuzzy set in a fts X. The interior of A is the union of all fuzzy open sets contained in A, denoted by $int(A)$. That is, $int(A) = \vee\{B : B$ is fuzzy open set , $B \leq A\}$.

Definition 2.10:[2] Let A be any fuzzy set in a fts X. The closure of A is the intersection of all fuzzy closed sets containing A, denoted by $cl(A)$. That is, $cl(A) = \wedge\{B : B$ is fuzzy closed set, $B \geq A\}$. The most important properties of the closure and interior of fuzzy sets are listed in the following proposition.

Proposition 2.11:[10] If A is any fuzzy set in X then:

i. A is a fuzzy open (closed) set if and only if $A = int(A)$ $(A = cl(A))$,
ii. $cl(1_X - A) = 1_X - int(A)$,
iii. $int(1_X - A) = 1_X - cl(A)$.

Proposition 2.12:[11] Let A,B be two fuzzy sets in a fts X. Then:

i. $int(A) \leq A$, $int(int(A)) = int(A)$,
ii. $int(A) \leq int(B)$, whenever $A \leq B$,
iii. $int(A \wedge B) = int(A) \wedge int(B)$, $int(A \vee B) \geq int(A) \vee int(B)$,
iv. $A \leq cl(A)$, $cl(cl(A)) = cl(A)$,
v. $cl(A) \leq cl(B)$, whenever A £ B,

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vi. $cl(A \wedge B) \leq cl(A) \wedge cl(B)$, $cl(A \vee B) = cl(A) \vee cl(B)$.

Definition 2.13:[1] Let (X, T) be a fuzzy topological space. Then X is called:

i. fuzzy T_0 space (FT_0 space, for short) iff for each pair of distinct fuzzy points in X, there exists an fuzzy open set in X containing one and not the other .

ii. fuzzy T_1 space (FT_1 space, for short) iff for each pair of distinct fuzzy points $x_\lambda$and y_α of X, there exists an fuzzy open sets G, H containing $x_\lambda$and y_α respectively such that $y_\alpha \notin G$ and $x_\lambda \notin H$.

iii. fuzzy T_2 space (FT_2 space, for short) iff for each pair of distinct fuzzy points $x_\lambda$and y_α of X, there exist disjoint fuzzy open sets G, H in X such that $x_\lambda \in G$ and $y_\alpha \in H$.

iv. fuzzy regular space iff for each fuzzy closed set A and for each $x_\lambda \notin A$, there exist disjoint fuzzy open sets G,H such that $x_\lambda \in G$ and A $\leq H$.

v. fuzzy normal space iff for each pair of disjoint fuzzy closed sets A and B, there exist disjoint

vi. fuzzy open sets G and Hsuch that A $\leq G$ and B $\leq H$.

$\hat{\mu}\beta$ KERNEL SET IN FUZZY TOPOLOGICAL SPACES

Definition 3.1: The intersection of all fuzzy $\hat{\mu}\beta$open subsets of a fuzzy topological space (X, T) containing A is called the$\hat{\mu}\beta$ kernel of A (briefly $\hat{\mu}\beta\ker(A)$), this means that $\hat{\mu}\beta\ker(A) = \wedge\{G \in T : A \leq G\}$.

Definition 3.2: In a fuzzy topological space (X, T), a set A is said to be weakly ultra $\hat{\mu}\beta$ separated from B if there exists a fuzzy $\hat{\mu}\beta$ open set G such that A $\leq G$ and G $\wedge B = 0_X$or

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$A \wedge \hat{\mu}\beta cl(B) = 0_X$.

Remark 3.3: By definition (3.2), we have the following: For every two distinct fuzzy points $x_\lambda$and y_α of X, $\hat{\mu}\beta$ker (x_λ) = { y_α : $\{x_\lambda\}$is not weakly ultra $\hat{\mu}\beta$separated from $\{y_\alpha\}$ }.

Definition 3.4: A fuzzy topological space (X, T) is called fuzzy $\hat{\mu}\beta R_0$ space ($F\hat{\mu}\beta R_0$ space, for short) if for each fuzzy $\hat{\mu}\beta$ open set U and$x_\lambda \in U$ then $\hat{\mu}\beta cl\ (x_\lambda) \leq U$.

Definition 3.5: A fuzzy topological space (X, T) is called fuzzy $\hat{\mu}\beta R_1$ space ($F\hat{\mu}\beta R_1$ space, for short) if for each two distinct fuzzy points $x_\lambda$and y_α of X with $\hat{\mu}\beta cl\{x_\lambda\} \neq \hat{\mu}\beta cl\{y_\alpha\}$, there exist disjoint fuzzy $\hat{\mu}\beta$ open sets U, V such that $\hat{\mu}\beta cl\{x_\lambda\} \leq U$ and $\hat{\mu}\beta cl\{x_\lambda\} \leq V$.

Remark 3.6: Each fuzzy separation axiom is defined as the conjunction of two weaker axioms: $F\hat{\mu}\beta T_1$ space = $F\hat{\mu}\beta R_{i-1}$ space and $F\hat{\mu}\beta T_{i-1}$space = $F\hat{\mu}\beta R_{i-1}$space and $F\hat{\mu}\beta T_0$ space, i = 1,2.

Lemma 3.7: Let (X, T) be a fuzzy topological space then $y_\alpha \in \hat{\mu}\beta$ ker$\{x_\lambda\}$iff $x_\lambda \in \hat{\mu}\beta cl\{y_\alpha\}$, for each x $\neq y \in X$.

Proof: Let $y_\alpha \notin \hat{\mu}\beta$ker $\{x_\lambda\}$, then there exist fuzzy open set V containing x_λ such that $y_\alpha \notin V$. Thus, $x_\lambda \notin \hat{\mu}\beta cl\{y_\alpha\}$. The converse part can be proved in a similar way.

Theorem 3.8: A fuzzy topological space (X, T)is $F\hat{\mu}\beta T_1$ space if and only if for each x $\neq y \in X, y_\alpha \notin \hat{\mu}\beta$ ker$\{x_\lambda\}$ and $x_\lambda \notin \hat{\mu}\beta$ker $\{y_\alpha\}$.

Proof: Let (X, T) be a $F\hat{\mu}\beta T_1$space then for each x $\neq y \in X$, there exists an fuzzy open sets U, V such that $x_\lambda \in U, y_\alpha \notin U$ and $y_\alpha \in V, x_\lambda \notin V$. Implies $y_\alpha \notin \hat{\mu}\beta$ker $\{x_\lambda\}$ and $x_\lambda \notin \hat{\mu}\beta$ker $\{y_\alpha\}$. Conversely, let $y_\alpha \notin \hat{\mu}\beta$ker $\{x_\lambda\}$ and $x_\lambda \notin \hat{\mu}\beta$ker $\{y_\alpha\}$, for each x $\neq y \in X$. Then there exists an fuzzy $\hat{\mu}\beta$ open sets U,V such that $x_\lambda \in U, y_\alpha \notin U$ and $y_\alpha \in V, x_\lambda \notin V$. Thus, (X, T) is a $F\hat{\mu}\beta T_1$space.

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Theorem 3.9: A fuzzy topological space (X, T) is space if and only if for each x $\in X$ then $\hat{\mu}\beta\ker\{x_\lambda\} = \{x_\lambda\}$.

Proof: Let (X, T) be a $F\hat{\mu}\beta T_1$ space and let $\hat{\mu}\beta\ker\{x_\lambda\} \neq \{x_\lambda\}$then $\hat{\mu}\beta\ker\{x_\lambda\}$contains another fuzzy point distinct from x_λ say y_α.So $y_\alpha \in \hat{\mu}\beta\ker\{x_\lambda\}$. Hence by theorem (3.8), (X, T) is not a $F\hat{\mu}\beta T_1$space this is a contradiction. Thus,$\hat{\mu}\beta\ker\{x_\lambda\} = \{x_\lambda\}$. Conversely, let $\hat{\mu}\beta\ker\{x_\lambda\} = \{x_\lambda\}$, for each x $\in X$ and let (X, T) be not a $F\hat{\mu}\beta T_1$ space. Then, by theorem (3.8) $y_\alpha \in \hat{\mu}\beta\ker\{x_\lambda\}$, implies $\hat{\mu}\beta\ker\{x_\lambda\} \neq \{x_\lambda\}$, this is a contradiction. Thus, (X, T)is a $F\hat{\mu}\beta T_1$ space.

Definition 3.10: Let (X, T) be a fuzzy topological space. A fuzzy point x_λ is said to be$\hat{\mu}\beta$ kernelled fuzzy point of A $\leq$X(Briefly $x_\lambda \in \hat{\mu}\beta\ker\{x_\lambda\}$) if and only if for each G fuzzy $\hat{\mu}\beta$ closed set contains then x_λ then G $\wedge$ A $\neq 0_X$.

Definition 3.11: Let (X, T) be a fuzzy topological space. A fuzzy point x_λ is said to be boundary $\hat{\mu}\beta$ kernelled fuzzy point of A (Briefly $x_\lambda \in \hat{\mu}\beta ker_{br}(A)$) if and only if for each fuzzy $\hat{\mu}\beta$closed set G contains x_λ then G$\wedge A \neq 0_X$ and G $\wedge$ $A^C \neq 0_X$.

Definition 3.12: Let (X, T)be a fuzzy topological space. A fuzzy point x_λ is said to be derived$\hat{\mu}\beta$ kernelled fuzzy point of A (Briefly $x_\lambda \in$ $\hat{\mu}\beta ker_{dr}(A)$) if and only if for each G fuzzy$\hat{\mu}\beta$ closed set contains x_λ then A$\wedge$ G / $\{x_\lambda\}$ $\neq$ 0_X.

Definition 3.13: By definition (3.10), we have the following: For every two distinct fuzzy points $x_\lambda$and y_α of X, $\hat{\mu}\beta\ker\{x_\lambda\}$= $\{y_\alpha : x_\lambda \in G_{y_\alpha}, {G_{y_\alpha}}^c \in T\}$.

Theorem 3.14: Let (X, T) be a fuzzy topological space and x $\neq y \in X$. Then x_λ is a $\hat{\mu}\beta$ kernelled fuzzy point of $\{y_\alpha\}$ if and only if $y_\alpha$is an $\hat{\mu}\beta$ adherent fuzzy point of $\{x_\lambda\}$.

Proof: Let x_λ be a$\hat{\mu}\beta$ kernelled fuzzy point of $\{y_\alpha\}$. Then for every fuzzy $\hat{\mu}\beta$ closed set G such that $x_\lambda \in G$ implies $y_\alpha \in G$, then $y_\alpha \in \wedge\{G: x_\lambda \in G\}$ this means $y_\alpha \in \hat{\mu}\beta cl\{x_\lambda\}$. Thus

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y_α is an $\hat{\mu}\beta$ adherent fuzzy point of $\{x_\lambda\}$. Conversely, let y_α be an $\hat{\mu}\beta$ adherent fuzzy point of $\{x_\lambda\}$. Then for every fuzzy $\hat{\mu}\beta$ open set U such that $y_\alpha \in U$ implies $x_\lambda \in U$, then $x_\lambda \in \wedge \{U: y_\alpha \in U\}$, this means $x_\lambda \in \hat{\mu}\beta\ker\{y_\alpha\}$. Thus, $x_\lambda$is a$\hat{\mu}\beta$ kernelled fuzzy point of $\{y_\alpha\}$.

Theorem 3.15: Let (X, T) be a fuzzy topological space and A$\leq X$ and let $\hat{\mu}\beta ker_{dr}(A)$be the set of all $\hat{\mu}\beta$kernelled derived fuzzy points of A, then $\hat{\mu}\beta\ker(A) = A \vee \hat{\mu}\beta ker_{dr}(A)$.

Proof: Let $x_\lambda \in A \vee \hat{\mu}\beta ker_{dr}(A)$and if $x_\lambda \in \hat{\mu}\beta ker_{dr}(A)$, then for every fuzzy $\hat{\mu}\beta$closed set G intersects A. Therefore, $x_\lambda \in \hat{\mu}\beta ker_{dr}(A)$. Hence, $\hat{\mu}\beta ker_{dr}(A) \leq \hat{\mu}\beta\ker(A)$, it follows That A $\vee \hat{\mu}\beta ker_{dr}(A) \leq \hat{\mu}\beta\ker(A)$. To demonstrate the reverse inclusion, we consider x_λ be a fuzzy point of $\hat{\mu}\beta\ker(A)$. If $x_\lambda \in A$, then $x_\lambda \in A \leq \hat{\mu}\beta ker_{dr}(A)$. Suppose that $x_\lambda \notin A$. Since $x_\lambda \in \hat{\mu}\beta\ker(A)$, then for every fuzzy $\hat{\mu}\beta$closed set G containing x_λ implies G$\wedge A \neq 0_X$, this means A$\wedge G/\{x_\lambda\} \neq 0_X$.Then,$x_\lambda \in \hat{\mu}\beta ker_{dr}(A)$, so that $x_\lambda \in A \vee \hat{\mu}\beta ker_{dr}(A)$.

Theorem 3.16: Let (X, T) be a fuzzy topological space and A$\leq X$ and let$\hat{\mu}\beta ker_{br}(A)$be the set of all $\hat{\mu}\beta$kernelled boundary fuzzy points of A, then $\hat{\mu}\beta\ker(A) = A \vee \hat{\mu}\beta ker_{br}(A)$.

Proof: Let $x_\lambda \in A \vee \hat{\mu}\beta ker_{br}(A)$ and if $x_\lambda \in \hat{\mu}\beta ker_{br}(A)$, then for every fuzzy $\hat{\mu}\beta$closed set G intersects A. Hence,$\hat{\mu}\beta ker_{br}(A) \leq \hat{\mu}\beta\ker(A)$, it follows that A $\vee \hat{\mu}\beta ker_{br}(A) \leq \hat{\mu}\beta\ker(A)$. To demonstrate the reverse inclusion, we consider $x_\lambda$be a fuzzy point of $\hat{\mu}\beta\ker(A)$. If $x_\lambda \in A$, then $x_\lambda \in A \vee \hat{\mu}\beta ker_{br}(A)$. Suppose that $x_\lambda \notin A$, implies $x_\lambda \in A^C$ Since $x_\lambda \in \hat{\mu}\beta\ker(A)$,then for every fuzzy $\hat{\mu}\beta$closed set G containing x_λ implies G$\wedge A \neq 0_X$ and G $\wedge A^c \neq 0_X$. Then $x_\lambda \in \hat{\mu}\beta ker_{br}(A)$, so that $x_\lambda \in A \vee \hat{\mu}\beta ker_{br}(A)$. Hence, $\hat{\mu}\beta\ker(A) \leq$.A $\vee \hat{\mu}\beta ker_{br}(A)$ Thus,$\hat{\mu}\beta\ker(A) = A \vee \hat{\mu}\beta ker_{br}(A)$.

Corollary 3.17: Every $\hat{\mu}\beta$ interior fuzzy point is a$\hat{\mu}\beta$ kernelled fuzzy point.

Proof: Since $\hat{\mu}\beta int(A) \leq A \leq \hat{\mu}\beta\ker(A)$. Thus, every $\hat{\mu}\beta$ interior fuzzy point is a$\hat{\mu}\beta$ kernelled fuzzy point.

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Theorem 3.18: Let (X, T) be a fuzzy topological space and A is a subset of X. Then A is a fuzzy$\hat{\mu}\beta$open set if and only if everyx_λ, $\hat{\mu}\beta$ kernelled fuzzy point of A is an $\hat{\mu}\beta$ interior fuzzy point of A.

Proof: Let A be a fuzzy$\hat{\mu}\beta$open set, then $\hat{\mu}\beta\ker(A) = A = \hat{\mu}\beta int(A)$, implies every$\hat{\mu}\beta$ kernelled fuzzy point is an $\hat{\mu}\beta$ interior fuzzy point. Conversely, let everyx_λ, $\hat{\mu}\beta$ kernelled fuzzy point of A is an$\hat{\mu}\beta$ interior fuzzy point of A. Then $\hat{\mu}\beta\ker(A) \leq \hat{\mu}\beta int(A)$. Hence, $\hat{\mu}\beta int(A) \leq A \leq \hat{\mu}\beta\ker(A)$, implies $\hat{\mu}\beta int(A) = A = \hat{\mu}\beta\ker(A)$. Thus A is a fuzzy$\hat{\mu}\beta$ open set.

Corollary 3.19: A subset A of X is a fuzzy open set if and only if for each x_λ, $\hat{\mu}\beta$kernelled fuzzy point then $x_\lambda \notin A^C$.

Proof: By theorem (3.18).

Theorem 3.20: A subset A of X is a fuzzy $\hat{\mu}\beta$ closed set if and only if $\hat{\mu}\beta\ker(A^C) \wedge \hat{\mu}\beta \mathrm{cl}(A) = 0_X$.

Proof: Let A is a fuzzy $\hat{\mu}\beta$ closed set. Then $A^C$is a fuzzy$\hat{\mu}\beta$ open set, implies $A^C = \hat{\mu}\beta\ker(A^C)$ by theorem(3.18). Hence A=$\hat{\mu}\beta cl(A)$. Thus $\hat{\mu}\beta\ker(A^C) \wedge \hat{\mu}\beta cl(A) = 0_X$. Conversely, letThus $\hat{\mu}\beta\ker(A^C) \wedge \hat{\mu}\beta cl(A) = 0_X$, then for each $x_\lambda \in \hat{\mu}\beta\ker(A^C)$, implies $x_\lambda \notin \hat{\mu}\beta cl(A)$. Therefore $x_\lambda \in \hat{\mu}\beta int(A^C)$. Hence by theorem (3.18), A^C is a fuzzy$\hat{\mu}\beta$ open set. Thus A is a$\hat{\mu}\beta$fuzzy closed set.

Theorem 3.21: A fuzzy topological space (X, T) is $F\hat{\mu}\beta R_0$space if and only if every$\hat{\mu}\beta$ adherent fuzzy point of $\{x_\lambda\}$ is a $\hat{\mu}\beta$ kernelled fuzzy point of $\{x_\lambda\}$.

Proof: Let (X, T) be an $F\hat{\mu}\beta R_0$space. Then, for each $x \in X$, $\hat{\mu}\beta\ker\{x_\lambda\} = \hat{\mu}\beta cl\{x_\lambda\}$by lemma (3.7). Thus,every $\hat{\mu}\beta$ adherent fuzzy point of $\{x_\lambda\}$ is a $\hat{\mu}\beta$ kernelled fuzzy point of $\{x_\lambda\}$.

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Conversely, let every $\hat{\mu}\beta$ adherent fuzzy point of $\{x_\lambda\}$ is a $\hat{\mu}\beta$ kernelled fuzzy point of $\{x_\lambda\}$ and let U $\leq X$ and $x_\lambda \in U$. Then $\hat{\mu}\beta cl\{x_\lambda\} \leq \hat{\mu}\beta \ker\{x_\lambda\}$ for each x$\in X$. Since $\hat{\mu}\beta \ker\{x_\lambda\} = \wedge \{U: U \in T, x_\lambda \in U\}$, implies $\hat{\mu}\beta cl\{x_\lambda\} \leq U$, for each U fuzzy $\hat{\mu}\beta$ open set contains x_λ. Thus, (X, T) is an $F\hat{\mu}\beta R_0$ space.

Theorem 3.22: A fuzzy topological space (X, T) is $F\hat{\mu}\beta T_0$ space if and only if for each x $\neq y \in X$ either x_λ is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{y_\alpha\}$ or y_α is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{x_\lambda\}$.

Proof: Let (X, T) be an $F\hat{\mu}\beta T_0$ space. Then for each x $\neq y \in X$ there exists a fuzzy $\hat{\mu}\beta$ open set U such that $x_\lambda \in U$, $y_\alpha \notin U$, implies $y_\alpha \in U^c$. Hence U^c is a fuzzy $\hat{\mu}\beta$ closed, then y_α is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{x_\lambda\}$. Thus either x_λ is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{y_\alpha\}$ or y_α is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{x_\lambda\}$.

Conversely, let for each x $\neq y \in X$, either x_λ is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{y_\alpha\}$ or y_α is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{x_\lambda\}$. Then there exist a fuzzy $\hat{\mu}\beta$ closed set G such that $x_\lambda \in G, G \wedge \{y_\alpha\} = 0_X$ or $y_\alpha \in G, G \wedge \{x_\lambda\} = 0_X$, implies $x_\lambda \notin G^c, y_\alpha \in G^c$ or $x_\lambda \in G^c, y_\alpha \notin G^c$. Hence G^c is a fuzzy $\hat{\mu}\beta$ open set. Thus, (X, T) is a $F\hat{\mu}\beta T_0$ space.

Theorem 3.23: A fuzzy topological space (X, T) is $F\hat{\mu}\beta T_1$ space if and only if $\hat{\mu}\beta ker_{dr}\{x_\lambda\} = 0_X$, for each x$\in X$.

Proof: Let (X, T) be an $F\hat{\mu}\beta T_1$ space. Then for each x$\in X$, $\hat{\mu}\beta \ker\{x_\lambda\} = \{x_\lambda\}$, by theorem (3.9). Since $\hat{\mu}\beta ker_{dr}\{x_\lambda\} = \hat{\mu}\beta \ker\{x_\lambda\} - \{x_\lambda\}$. Thus, $\hat{\mu}\beta ker_{dr}\{x_\lambda\} = 0_X$. Conversely, let $\hat{\mu}\beta ker_{dr}\{x_\lambda\} = 0_X$. By theorem (3.14), $\hat{\mu}\beta \ker\{x_\lambda\} = \{x_\lambda\} \vee \hat{\mu}\beta ker_{dr}\{x_\lambda\}$, implies $\hat{\mu}\beta \ker\{x_\lambda\} = \{x_\lambda\}$. Hence, by theorem (3.9), (X, T) is a $F\hat{\mu}\beta T_1$ space.

Theorem 3.24: A fuzzy topological space (X, T) is $F\hat{\mu}\beta T_1$ space if and only if for each x $\neq y \in X$, x_λ is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{y_\alpha\}$ and y_α is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{x_\lambda\}$.

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Proof: Let (X, T) be a $F\hat{\mu}\beta T_1$ space. Then for each x $\neq y \in X$ there exist fuzzy $\hat{\mu}\beta$ open sets U,V suchthat$x_\lambda \in U, y_\alpha \notin U$ and $y_\alpha \in V, x_\lambda \notin V$ implies $x_\lambda \in V^c, \{y_\alpha\} \wedge V^c = 0_Xandy_\alpha \in U^c$,$\{x_\lambda\} \wedge U^c = 0_X$. Hence U^c and V^c are fuzzy $\hat{\mu}\beta$ closed sets. Thus x_λ is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{y_\alpha\}$ and $y_\alpha$is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{x_\lambda\}$.Conversely, let for each x $\neq y \in X$ not $\hat{\mu}\beta$ kernelled fuzzy point of $\{y_\alpha\}$ and y_α is not $\hat{\mu}\beta$ kernelled fuzzy point of $\{x_\lambda\}$. Then, there exist fuzzy $\hat{\mu}\beta$ closed sets G_1, G_2 such that $x_\lambda \in G_1, G_1 \wedge \{y_\alpha\} = 0_X$ and $y_\alpha \in G_2, G_2 \wedge \{x_\lambda\} = 0_X$, implies $x_\lambda \in {G_2}^c, y_\alpha \notin {G_2}^c$and $y_\alpha \in {G_1}^c, x_\lambda \notin {G_1}^c$. Hence ${G_1}^c$and${G_2}^c$are fuzzy $\hat{\mu}\beta$
open sets. Thus,(X, T) is $F\hat{\mu}\beta T_1$ space.

4. $\hat{\mu}\beta kr$- Spaces in Fuzzy Topological Spaces:

Definition 4.1: A fuzzy topological space (X, T) is said to be a *kr* space if and only if for each subset A of X then, ker(A) is a fuzzy open set.

Definition 4.2: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is called fuzzy $\hat{\mu}\beta T_k$ space ($F\hat{\mu}\beta T_k$ space, for short) if and only if for each $x_\lambda \in X$, then $\hat{\mu}\beta ker_{dr}\{x_\lambda\}$ is a fuzzy $\hat{\mu}\beta$ open set.

Theorem 4.3: In fuzzy topological $\hat{\mu}\beta kr$ space (X, T), every $F\hat{\mu}\beta T_1$ space is a $F\hat{\mu}\beta T_k$ space.

Proof: Let (X, T)be a $F\hat{\mu}\beta T_1$ space. Then, for each $x_\lambda \in X, \hat{\mu}\beta ker$ $\{x_\lambda\}$ =$\{x_\lambda\}$ by theorem (3.9).

As $\hat{\mu}\beta ker_{dr}\{x_\lambda\} = \hat{\mu}\beta \ker\{x_\lambda\} - \{x_\lambda\}$, implies $\hat{\mu}\beta ker_{dr}\{x_\lambda\} = 0_X$. Thus, (X, T) is a $F\hat{\mu}\beta T_k$ space.

Theorem 4.4: In fuzzy topological $\hat{\mu}\beta kr$ space (X, T), every $F\hat{\mu}\beta T_k$ space is a $F\hat{\mu}\beta T_0$space.

Proof: Let (X, T) be a $F\hat{\mu}\beta T_k$ space and let x $\neq y \in X$. Then, $\hat{\mu}\beta ker_{dr}\{x_\lambda\}$ is a fuzzy $\hat{\mu}\beta$ open set, therefore,there exist two cases:_

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i. $y_\alpha \in \hat{\mu}\beta ker_{dr}\{x_\lambda\}$ is a fuzzy$\hat{\mu}\beta$ open set. Since $x_\lambda \notin \hat{\mu}\beta ker_{dr}\{x_\lambda\}$ Thus (X, T) is a $F\hat{\mu}\beta T_0$space.

ii. $y_\alpha \notin \hat{\mu}\beta ker_{dr}\{x_\lambda\}$, implies $y_\alpha \notin \hat{\mu}\beta$ker $\{x_\lambda\}$. But $\hat{\mu}\beta$ker $\{x_\lambda\}$ is a fuzzy $\hat{\mu}\beta$ open set. Thus (X, T) is a $F\hat{\mu}\beta T_0$space.

Definition 4.5: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is said to be fuzzy $\hat{\mu}\beta T_L$ space ($F\hat{\mu}\beta T_L$ space, forshort) if and only if for each x $\neq y \in X$ is degenerated (empty or singleton fuzzy set).

Theorem 4.6: In fuzzy topological $\hat{\mu}\beta kr$ space (X, T), every $F\hat{\mu}\beta T_1$ space is $F\hat{\mu}\beta T_L$ space.

Proof: Let (X, T) be a $F\hat{\mu}\beta T_L$ space. Then for each x $\neq y \in X$ and $\hat{\mu}\beta ker$ $\{x_\lambda\} = \{x_\lambda\}$ and $\hat{\mu}\beta$ ker$\{y_\alpha\} = \{y_\alpha\}$by theorem (3.9), $\hat{\mu}\beta$ ker$\{x_\lambda\} \wedge \hat{\mu}\beta$ ker$\{y_\alpha\} = 0_X$. Thus (X, T) is a $F\hat{\mu}\beta T_L$ space.

Theorem 4.7: In fuzzy topological $\hat{\mu}\beta kr$ space (X, T), every $F\hat{\mu}\beta T_L$ space is a $F\hat{\mu}\beta T_0$ space.

Proof: Let (X, T) be a $F\hat{\mu}\beta T_L$ space. Then for each x$\neq y \in X, \hat{\mu}\beta$ker $\{x_\lambda\} \wedge \hat{\mu}\beta$ker $\{y_\alpha\}$ is degenerated(empty or singleton fuzzy set). Therefore there exist three cases:

i. $\hat{\mu}\beta$ ker$\{x_\lambda\} \wedge \hat{\mu}\beta$ ker$\{y_\alpha\} = 0_X$implies (X, T) is a $F\hat{\mu}\beta T_0$ space.

ii. $\hat{\mu}\beta$ ker$\{x_\lambda\} \wedge \hat{\mu}\beta$ ker$\{y_\alpha\} = \{x_\lambda\} or$ $\{y_\alpha\}$implies $y_\alpha \notin$ $\hat{\mu}\beta$ker $\{x\}$ or $x_\lambda \notin \hat{\mu}\beta$ker $\{y_\alpha\}$ implies (X, T)is a$F\hat{\mu}\beta T_0$space.

iii. $\hat{\mu}\beta$ ker$\{x_\lambda\} \wedge \hat{\mu}\beta$ ker$\{y_\alpha\} = \{z_\beta\}$, z $\neq x \neq y, z \in X$, implies $y_\alpha \notin \hat{\mu}\beta$ker $\{x_\lambda\}$ and $x_\lambda \notin \hat{\mu}\beta$ker $\{y_\alpha\}$,implies (X, T) is a $F\hat{\mu}\beta T_0$ space.

Definition 4.8: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is said to be a fuzzy $\hat{\mu}\beta T_N$ space ($F\hat{\mu}\beta T_N$ space, forshort) if and only if for each x$\neq y \in X, \hat{\mu}\beta$ ker$\{x_\lambda\} \wedge \hat{\mu}\beta$ ker$\{y_\alpha\}$ is empty or $\{x_\lambda\}$ or $\{y_\alpha\}$.

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Theorem 4.9: In fuzzy topological $\hat{\mu}\beta kr$ space (X, T), every $F\hat{\mu}\beta T_1$ space is $F\hat{\mu}\beta T_N$space.

Proof: Let (X, T) be a $F\hat{\mu}\beta T_N$ space. Then for each x $\neq y \in X, \hat{\mu}\beta \ker\{x_\lambda\} = \{x_\lambda\}$ and$\hat{\mu}\beta \ker\{y_\alpha\} = \{y_\alpha\}$ theorem (3.9), implies$\hat{\mu}\beta \ker\{x_\lambda\} \wedge \hat{\mu}\beta \ker\{y_\alpha\} = 0_X$. Thus (X, T) is a $F\hat{\mu}\beta T_N$ space.

Theorem 4.10: In fuzzy topological $\hat{\mu}\beta kr$ space (X, T), every $F\hat{\mu}\beta T_N$ space is a $F\hat{\mu}\beta T_0$ space.

Proof: Let (X, T) be a $F\hat{\mu}\beta T_N$ space. Then for each x $\neq y \in X$ is degenerated (empty or singleton fuzzy set). Therefore there exist two cases:_

i. $\hat{\mu}\beta \ker\{x_\lambda\} \wedge \hat{\mu}\beta \ker\{y_\alpha\} = 0_X$, implies (X, T) is a $F\hat{\mu}\beta T_0$ space.

ii. $\hat{\mu}\beta \ker\{x_\lambda\} \wedge \hat{\mu}\beta \ker\{y_\alpha\} = \{x_\lambda\}$ or $\{y_\alpha\}$ implies $\{y_\alpha\} \notin \hat{\mu}\beta\ker \{x_\lambda\}$ or $x_\lambda \notin \hat{\mu}\beta\ker \{y_\alpha\}$, implies (X, T)is a$F\hat{\mu}\beta T_0$ space.

Theorem 4.11: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is $F\hat{\mu}\beta T_2$ space iff for each x $\neq y \in X$,, then$\hat{\mu}\beta \ker\{x_\lambda\} \wedge \hat{\mu}\beta \ker\{y_\alpha\} = 0_X$.

Proof: Let a fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is $F\hat{\mu}\beta T_2$ space. Then for each x $\neq y \in X$, there existdisjoint fuzzy $\hat{\mu}\beta$open sets U,V such that $x_\lambda \in U$, and $y_\alpha \in V$. Hence $\hat{\mu}\beta \ker\{x_\lambda\} \leq$U and $\hat{\mu}\beta \ker\{y_\alpha\} \leq$V.Thus $\hat{\mu}\beta \ker\{x_\lambda\} \wedge \hat{\mu}\beta \ker\{y_\alpha\} = 0_X$.Conversely, let for each x $\neq y \in X, \hat{\mu}\beta \ker\{x_\lambda\} \wedge \hat{\mu}\beta \ker\{y_\alpha\} = 0_X$. Since (X, T) is a fuzzy topological$\hat{\mu}\beta kr$ space. Thus (X, T) is $F\hat{\mu}\beta T_2$space.

Theorem 4.12: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is fuzzy regular space iff for each G fuzzy$\hat{\mu}\beta$ closed set and $x_\lambda \notin G$, then $\hat{\mu}\beta \ker\{x_\lambda\} \wedge \hat{\mu}\beta \ker\{y_\alpha\} = 0_X$.

Proof: By the same way of proof of theorem (4.11).

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Theorem 4.13: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is fuzzy normal space iff for each disjoint fuzzy$\hat{\mu}\beta$closed sets G,H, then $\hat{\mu}\beta \ker\{x_\lambda\} \wedge \hat{\mu}\beta \ker\{y_\alpha\} = 0_X$.

Proof: By the same way of proof of theorem (4.11).

Theorem 4.14: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is $F\hat{\mu}\beta T_1$space iff it is $F\hat{\mu}\beta R_0$ space and $F\hat{\mu}\beta T_K$ space.

Proof: It follows from theorem (4.3) and remark (3.6).

Theorem 4.15: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is $F\hat{\mu}\beta T_1$ space iff it is $F\hat{\mu}\beta R_0$ space and $F\hat{\mu}\beta T_L$space.

Proof: It follows from theorem (4.6) and remark (3.6).

Theorem 4.16: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is $F\hat{\mu}\beta T_1$space if and only if it is $F\hat{\mu}\beta R_0$space and$F\hat{\mu}\beta T_N$ space.

Proof: It follows from theorem (4.9) and remark (3.6)._

Theorem 4.17: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is $F\hat{\mu}\beta T_i$space if and only if it is $F\hat{\mu}\beta R_{i-1}$ space and $F\hat{\mu}\beta T_K$space, i=12.

Proof: It follows from theorem (4.3) and remark (3.6).

Theorem 4.18: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is $F\hat{\mu}\beta T_i$ space if and only if it is $F\hat{\mu}\beta R_{i-1}$ space and $F\hat{\mu}\beta T_L$space, i= 1, 2.

Proof: It follows from theorem (4.6) and remark (3.6).

Theorem 4.19: A fuzzy topological $\hat{\mu}\beta kr$ space (X, T) is $F\hat{\mu}\beta T_i$ space if and only if it is $F\hat{\mu}\beta R_{i-1}$ space and $F\hat{\mu}\beta T_N$space, i = 1,2.

Proof: It follows from theorem (4.9) and remark (3.6)._

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Remark 4.20: The relation between fuzzy $\hat{\mu}\beta$ separation axioms can be representing as a matrix.

Therefore, the element a_ij refers to this relation. As the following matrix representation shows

and	$F\hat{\mu}\beta T_0$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta R_0$	$F\hat{\mu}\beta R_1$	$F\hat{\mu}\beta T_K$	$F\hat{\mu}\beta T_L$	$F\hat{\mu}\beta T_N$
$F\hat{\mu}\beta T_0$	$F\hat{\mu}\beta T_0$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_K$	$F\hat{\mu}\beta T_L$	$F\hat{\mu}\beta T_N$
$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_1$
$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$
$F\hat{\mu}\beta R_0$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta R_0$	$F\hat{\mu}\beta R_1$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_1$
$F\hat{\mu}\beta R_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta R_1$	$F\hat{\mu}\beta R_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_2$
$F\hat{\mu}\beta T_K$	$F\hat{\mu}\beta T_K$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_K$	$F\hat{\mu}\beta T_L$	$F\hat{\mu}\beta T_0$
$F\hat{\mu}\beta T_L$	$F\hat{\mu}\beta T_L$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_L$	$F\hat{\mu}\beta T_L$	$F\hat{\mu}\beta T_0$
$F\hat{\mu}\beta T_N$	$F\hat{\mu}\beta T_N$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_1$	$F\hat{\mu}\beta T_2$	$F\hat{\mu}\beta T_0$	$F\hat{\mu}\beta T_0$	$F\hat{\mu}\beta T_N$

Matrix Representation

The relation between fuzzy μ̓β separation axioms in fuzzy topological μ̓βkr spaces

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