MANTECH Publications

Journal of Engineering Mathematics & Statistics
Volume 3, Issue 1, January-June 2019

Symmetric Bi-multipliers on BCK-algebra

J. Goldarexy[1], M. Indhumathi[2]

II M.Sc.[1], Associate Professor[2]

Department of Mathematics

Rathnavel Subramanian college of Arts and Science, Sulur

Corresponding authors' email id: *rexyjesukumar@gmail.com[1], indumathi@rvsgroup.com[2]*

Abstract

In this paper, we introduce the notion of symmetric bi-multipliers in BCK-algebras. Also, we investigate some of their relative properties. Moreover, we defined kernel and fixed set of BCK-algebras and obtain some properties with illustrations.

Keywords: *BCK-algebras, multipliers, fixed set, kernel.*

INTRODUCTION

Imai and Iseki introduced two classes of abstract algebra: BCK-algebras and BCI-algebras [2] and [3]. The class of BCK-algebras is a proper subclass of the class of BCI-algebras. It is known that the notion of BCI-algebras is a generalization of BCK-algebras, and investigated relations between d-algebras and BCK-algebras.

The notion of multipliers on lattices was introduced and studied by [6] and it was generalized to the partial multipliers on partially ordered sets in [7, 8]. Muhammad Anwar Chaudhry and Faisal Ali defined the notion of multipliers on d-algebras in [1].

In this paper the notion of symmetric bi-multipliers in BCK-algebras are given and properties of these multipliers are researched. Also, kernels and set of fixed points of a BCK-algebra are characterized by symmetric bi-multipliers.

PRELIMINARIES

In this section, we recall some basic definitions and its properties that are needful for developing the main results.

Definition2.1. [9] Let X be a set with a binary operation $*$ and a constant 0. Then $(X,*,0)$ of type $(2,0)$ is called a *BCK-*

Page 73-80 © MANTECH PUBLICATIONS 2019. All Rights Reserved

algebras if it's satisfied the following conditions, for all $x, y, z \in X$.

(i) $\big((x * y) * (x * z)\big) * (z * y) = 0$
(ii) $\big(x * (x * y)\big) * y = 0$
(iii) $x * x = 0$
(iv) $x * y = 0$ and $y * x = 0$ imply $x = y$
(v) $0 * x = 0$.

Example 2.2. Let $X = \{0, a, b\}$ and with the binary operation $*$defined by

$*$	**0**	***a***	***b***
0	0	0	0
a	*a*	0	0
b	*b*	b	0

Then X is a *BCK-algebra*.

Definition2.3. [5] Let*S* be a non-empty subset of a *BCK-algebrasX*. Then *S* is called *sub algebra* of *X* if$x * y \in S$ for all $x, y \in S$.

Definition2.4.[9]Let *X* be a *BCK-algebra* and I be a subset of *X*, then I is called an *ideal* of *X* if it satisfies the following conditions.

(i) $0 \in I$
(ii) $x * y \in I$ and $y \in I$ imply $x \in I$.

Definition2.5. [9] Let *X* be a *BCK*-algebra and *I* be a non empty subset of *X*, then *I* is called a *BCK*-ideal of *X* if it satisfies the following conditions.

(i) $x * y \in I$ and $y \in I$ imply $x \in I$ and
(ii) $x \in I$ and $y \in X$ imply $x * y \in I$ from condition
(iii) It is obvious that for $x \in I \subseteq X$: $0 = x * x \in I$.

A non empty subset I of X is called.

- A *commutative ideal* of *X*, if$0 \in I$ and if $(x * y) * z \in I$ and $z \in I$imply that
  $x * (y * (y * x)) \in I$For all $x, y, z \in X$.

- An *implicative ideal* of *X*, if $0 \in I$ and $\big(x * (y * z)\big) * z \in I$ and $z \in I$ imply that $x \in I$, for all $x, y, z \in X$.

Page 73-80 © MANTECH PUBLICATIONS 2019. All Rights Reserved

- A *commutative ideal* of *X*, if$0 \in I$ and if $(x * y) * z \in I$ and $z \in I$imply that $x * (y * (y * x)) \in I$For all $x, y, z \in X$.

- An *implicative ideal* of *X*, if $0 \in I$ and $\big(x * (y * z)\big) * z \in I$ and $z \in I$ imply that $x \in I$, for all $x, y, z \in X$.

- A *positive implicative ideal* of *X*, if $0 \in I$ and if $(x * y) * z \in I$ and $y * z \in I$ imply that $x * z \in I$, for all $x, y, z \in X$.

- A *maximal ideal* of *X*, if *I* is a proper ideal of *X* and not a proper subset of any proper ideal of *X*.

- An *obstinate ideal* of *X*, if *I* is a proper ideal of *X* and if $x, y \notin I$, implies $x * y \in I$ and $y * x \in I$, for all $x, y \in X$.

- An *irreducible ideal* of*X*, if I is an ideal of *X* and if $I = J \cap K$ implies $I = J$ or $I = K$ for ideals *J*,*K*, of *X*.

- A *normal ideal* of *X*, if *I* is an ideal of *X* and $x * (x * y) \in I$ implies $y * (y * x) \in I$, for all $x, y \in X$.

- A *varlet ideal* of *X*, if (1) $x \in I$ and $y \leq x$imply $y \in I$, (2) $x \in I$ and $y \in I$ imply that there exists $z \in I$ such that $x \leq z$ and $y \leq z$.

MAIN RESULTS

Symmetric bi-multipliers on BCK-algebras

In this section, we define the notion of symmetric bi-multipliers for *BCK-algebras* and obtain some useful results with illustrations.

Definition3.1.1. Let*X* be a *BCK*-algebra. A mapping $f(.,.):X \times X \to X$ is called *symmetric* if $f(x, y) = f(y, x)$for all $x, y \in X$.

Definition 3.1.2. Let X be a BCK-algebra and let f(.,.):X×X→X be a symmetric

Page 73-80 © MANTECH PUBLICATIONS 2019. All Rights Reserved

mapping. We call f symmetric bi-multipliers on X if it satisfies.

f(x,y*z)=f(x,y)*zFor all x,y,z∈X.

Example 3.1.3. Let X={0,a,b,c} and with the binary operation *defined by

*	**0**	***a***	***b***	***c***
0	0	0	0	0
a	*a*	0	0	*a*
b	*b*	*a*	0	b
c	*c*	*c*	c	0

Then*X* is *BCK-algebra*.

The mapping $f(.,.):X \times X \to X$ defined by

$$f(x,y) = \begin{cases} a, \text{if } x = y = b \\ 0, \text{otherwise.} \end{cases}$$

Then we can see that *f* is *symmetric bi-multipliers* on *X*.

Remark3.1.4: If *X* is a *BCK*-algebra with a binary operation $*$, then we can define a binary operation $\leq$ on *X* by;

$x \leq y$If and only if $x * y = 0$ for all $x, y \in X$.

Proposition3.1.5. Let *X* be a *BCK-algebra* and f be the symmetric bi-multiplier on *X*. then the followings hold for all $x, y, z \in X$.

$f(0,0) = 0$

$f(0,x) \leq x$

If $x \leq y$ then $f(0,x) \leq y$

Proof: Let X be a BCK-algebra and f be the symmetric bi-multiplier on X.

By using the definition of symmetric bi-multiplier on X and (I) We have the following:

f(0,0)=f(0,0*f(0,0))
=f(0,0)*f(0,0)=0
Therefore, f(0,0)=0

1. By (i) we have,0=f(0,0)=f(0,x*x)
=f(0*x)x

2. Therefore, we have f(0,x)*x=0 and hence f(0,x)≤x

3. Let x,y be elements in X and x≤y.
0=f(0,0)=f(0,x*y)=f(0,x)*y.
Therefore, we get 0=f(0,x)*y and hence f(0,x)≤y.

Definition3.1.6. A *BCK-algebraX* is said to be *positive implicative* If$(x * y) * z = (x * z) * (y * z)$ for all

Page 73-80 © MANTECH PUBLICATIONS 2019. All Rights Reserved

$x, y, z \in X$. Let *S(X)* be the collection of all symmetric bi-multipliers on *X*. It is clear that $O(.,.): X \times X \to X$ defined by $O(x,y) = 0$ for all $(x,y) \in X \times X$ and $P(.,.): X \times X \to X$ defined by $P(x,y) = x$ for all $(x,y) \in X \times X$are in *S(X)*. Therefore, *S(X)* is non empty.

Example 3.1.7. Consider a set $X = \{0, a, b\}$and with the binary operation $*$ is defined by

$*$	**0**	a	b
0	0	0	0
a	a	0	0
b	b	a	0

Then X is a positive implicative.

Definition3.1.8. Let X be a positive implicative BCK-algebra and S(X) be the collection of all symmetric bi-multipliers on X. we define a binary operation * on S(X)

〖 (f*g) 〗 _ ((x,y))=f(x,y)*g(x,y)

(x,y)∈X×XAndf,g∈S(X).

Example 3.1.9. Consider a set $X = \{0, a, b\}$and with the binary operation $*$ is defined by,

$*$	**0**	a	b
0	0	0	0
a	a	0	0
b	b	a	0

Then $X$is *symmetric bi-multipliers*on X.

Theorem3.2.0. Let X be a *positive implicativeBCK-algebra*. Then $(S(X), *, 0)$ is a *positive implicative BCK-algebra*.

Proof: Let X be a *positive implicative BCK-algebra* and let $g, f \in S(X)$

$$(g * f)_{(x,y)*(z,t)} = (g * f)_{(x,z)*(y,t)}$$

$$=$$

$$\big(g(x * z, y * t)\big) * \big(f(x * z, y * t)\big)$$

$$=$$

$$(g(x * z, y) * t) * (f(x * z, y) * t)$$

$$=$$

$$\big(g(x * z, y)\big) * \big(f(x * z, y)\big) * t$$

$$=(g * f)_{(x*z,y)} * t$$

So $g * f \in S(X)$.

Let $f \in S(X)$.Then

$$(O * f)_{(x,y)} = O(x,y) * f(x,y) = 0 * f(x,y) = 0 = O(x,y)$$

For all $(x,y) \in X \times X$ So $O * f = O$ for all $f \in S(X)$

Page 73-80 © MANTECH PUBLICATIONS 2019. All Rights Reserved

Now let $f \in S(X)$, we have

$(f * f_{(x,y)}) = f(x,y) * f(x,y) = 0 = O(x,y)$

For all $(x,y) \in X \times X$. So $f * f = O$.

Let $f, g \in S(X)$ such that $f * g = O$ and $g * f = O$.This implies that $(f * g)_{(x,y)} = 0$ and $(g * f)_{(x,y)} = 0$for all$(x,y) \in X \times X$. That is$f_{(x,y)} * g_{(x,y)} = 0$ and$g_{(x,y)} * f_{(x,y)} = 0$ which implies that$f_{(x,y)} = g_{(x,y)}$ for all $(x,y) \in X \times X$. Thus $f = g$. Hence $S(X)$ is a *BCK-algebra*. Now we need to show that it is *positive implicative*. Let $f, g, h \in S(X)$.Then

$((f * g) * h)_{(x,y)} = (f * g)_{(x,y)} * h(x,y)$

$=$

$\big(f(x,y) * g(x,y)\big) * h(x,y)$

$=$

$\big(f(x,y) * h*(x,y)\big) * (g(x,y) * h(x,y))$

$= ((f * h)_{(x,y)}) * ((g * h)_{(x,y)})$

$= ((f * h) * (g * h))_{(x,y)}$

For all$(x,y) \in X \times X$. Hence $(f * g) * h = (f * h) * (g * h)$ for all$f, g, h \in S(X)$. Therefore $S(X)$is an *implicative BCK-algebra*.

Definition3.2.1. Let f be *symmetric bi-multipliers*on X.We defined *Ker (f)* by *Ker(f)* $=\{x \in X / f(0,x) = 0$,forall$x \in X\}$.

Proposition3.2.2. Let *X* be a *BCK-algebra* and f be the *symmetric bi-multiplier* on *X*. Let $y \in Ker(f)$. Then we have $f(0,x) = 0$ and $f(0,y) = 0$.

So $f(0.x * y) = f(0,x) * y = 0 * \ y = 0$. Thus $x * y \in Ker(f)$. Therefore, *Ker (f)* is sub algebra of X.

Proof: Let X be a *BCK-algebra* and f be the *symmetric bi-multiplier* on X. let $x, y \in \ker(f)$. Then we have $f(0,x) = 0$ and $f(0,y) = 0$.

So $f(0, x * y) = f(0,x) * y = 0 * y = 0$. Thus $x * y \in \ker(f)$.therefore, $\ker(f)$ is *sub algebra* of X.

Definition3.2.3. A *BCK-algebraX*is called *commutative* if $x * (x * y) = y * (x * y)$ for all $(x,y) \in X$.

Example 3.2.4. Consider a set $X = \{0, a, b\}$ and with the binary operation $*$ is defined by

Page 73-80 © MANTECH PUBLICATIONS 2019. All Rights Reserved

*	**0**	***a***	***b***
0	0	0	0
a	a	0	0
b	b	a	0

Then X is *commutative.*

Proposition 3.2.5. Let X be a *commutative BCK-algebra* satisfying $x * 0 = 0, x \in X$ and f be the *symmetric bi-multiplier* on *X*. If$x \in Ker(f)$ and $y \leq x$ then $y \in Ker(f)$.

Proof: Let $x \in Ker(f)$ and $y \leq x$. Then we have $f(0,x) = 0$ and $y * x = 0$.and then

$$f(0,y) = f(0,y*0) = f(0,y*(y*x))$$
$$= f(0,x*(x*y))$$
$$= f(0,x)*(x*y)$$
$$= 0*(x*y)$$
$$= 0$$

Therefore $x \in Ker(f)$

Definition3.2.6. Let X be a *BCK-algebra* and f be the *symmetric bi-multiplier* on *X*.

Then the set

$Fix(f) = \{x \in X / f(0,x) = x\}$.

For all $x \in X$ is called the set of fixed points of f.

Proposition3.2.7. Let X be a *BCK-algebra* and f be the *symmetric bi-multiplier* on$X$. Then $Fix(f)$is sub algebra of$X$.

Proof: Let X be a *BCK-algebra* and$f$ be the *symmetric bi-multiplier* on X.

Since$f(0,0) = 0$ $Fix(f)$is non-empty. Let $x, y \in Fix(f)$. Then we have

$f(0,x) = x, f(0,y) = y$.

Then $f(0,x*y) = f(0,x)*y = x*y$.

Therefore $x*y \in Fix(f)$. Hence, $Fix(f)$ is sub algebraof X.

CONCLUSION

In this paper, We have introduced the notions of *BCK-algebras* and discussed some of their properties with examples. Further, we have defined the kernel and fixed set of *BCK-algebras.* Then, we have obtained some properties with illustrations.

REFERENCES

I. Chaudhry, M.A., and Ali, F.,*multipliers in d-algebras*, World applied Science Journal 18(11):1649-1653,(2012).
II. Isaki, K., *on BCI-algebras math*. Seminar Notes, 8 (1980), PP. 125130.

Page 73-80 © MANTECH PUBLICATIONS 2019. All Rights Reserved

III. Isaki, K., Tanaka, S.,*An introduction to theory of d-algebras*, Math. Japonica, 23(1978),PP.126.

IV. Larsen, R.,*An introduction to the theory of multipliers*, Berlin: Splinger-Verlag, (1971).

V. Neggers, J., and Kim, H.S.,*on d-algebras*, Math. Slovaca, Co., 49 (1999), 19-26.

VI. Szasz, G., *Derivations of Lattices*, Acta Sci. Math.(Szeged)37 (1975), 149-154.

VII. Szaz, A.,*partial Multipliers on partialordered sets*, Novi Sad J. Math. 32(1)(2002), 25-45.

VIII. Szaz, A., and Turi, J., *Characterizations of Injective Multipliers on Partially Ordered Sets*, studia Univ. "Babe-Bolyai" Mathematica XL, VII(1) (2002), 105-118.

IX. Tayebi Khorami, R., and Borumand Saeid, A., *Introduced in a new ideal of BCK-algebras, Fasc.Mathematica*, Tom XXII (2015), issue no: 1, 181-188.

Page 73-80 © MANTECH PUBLICATIONS 2019. All Rights Reserved