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On Dominator Coloring of Lollipop Graph

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Abstract

A graph has a dominator coloring if it has a proper coloring in which each vertex of the graph dominates every vertex of some color class. The dominator chromatic number χd (G) is the minimum number of colors required for a dominator coloring of G. In this paper the dominator chromatic number for Lollipop graph is studied and also the relation between dominator chromatic number, domination number and chromatic number is shown.

Keywords: *Coloring, Domination, Dominator Coloring, Lollipop graph*
Classification number: *05C15, 05C75*

INTRODUCTION

Let G= (V, E) be a graph such that V is the vertex set and E is the edge set. A dominating set S is a subset of the vertex set V of graph G such that every vertex in the graph either belongs to S or adjacent to S. The domination number γ (G) is the minimum cardinality of a dominating set of G.

A proper coloring of a graph G is a function from the set of vertices of a graph to a set of colors such that any two adjacent vertices have different colors. A subset of vertices colored with the same color is called a color class. The chromatic number is the minimum number of colors needed in a proper coloring of a graph and is denoted by χ (G).

A dominator coloring of a graph G is a proper coloring of graph such that every vertex of V dominates all vertices of at least one color class (possibly its ownclass). i.e., it is coloring of the vertices of a graph such that every vertex is either

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alone in its color class or adjacent to all vertices of at least one other class and this concept was introduced by Ralucca Michelle Gera in 2006 [2].

The dominator chromatic number χd (G) is the minimum number of color classes in a dominator coloring of G. The relation between dominator chromatic number, chromatic number and domination number of some classes of graphs were studied in [1], [3]. The dominator coloring of bipartite graph, star and double star graphs, central and middle graphs, fan, double fan, helm graphs etc. were also studied in various papers [4], [6], [7], [8].
In this paper the dominator chromatic number for Lollipop graph is studied and also the relation between dominator chromatic number, domination number and chromatic number is shown.

STRUCTURAL PROPERTIES OF LOLLIPOP GRAPH:

- Number of vertices in $L_{m,n}$ is m+ n
- Number of edges in $L_{m,n}$ is m+ n
- The Maximum degree in $(L_{m,n})$ is 3
- The Minimum degree in $(L_{3,n})$ is 1

DOMINATOR COLORING OF LOLLIPOP GRAPH:

Theorem 3.1:

The dominator chromatic number for ($L_{3,n}$),is $\chi_D([(L_{m,n})])= m$

Proof:

The ($L_{3,n}$) Lollipop graph is the graph obtained by joining a complete graph K_3 and the path P_n with a bridge.

Let V= { v_1 ,v_2, v_3,……., v_{m+n} } be the vertices of lollipop graphNow we assign the coloring to these vertices as follows:

Then the procedure to obtain dominator coloring of $L_{m.n}$ as follows:

The vertices of the complete graph are colored with the colors {c_1, c_2, ,…… c_m} (clockwise)

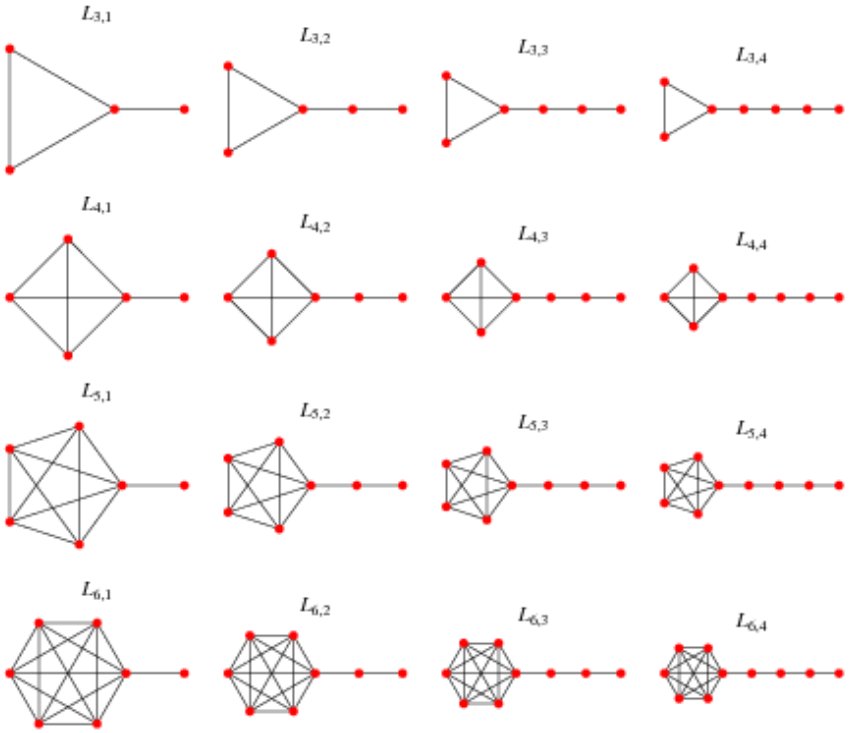
Assign colors 1 & 2 alternatively to the remaining (path) of the bridge
By the Definition of dominator coloring, the vertex set V dominates the color class.

Combining these colors we required m colors for dominator coloring.
Clearly, the above said coloring is dominatic and is minimum.

Hence $\chi_D([(L_{m,n})]) = m$

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Lollipop Graph



Example 3.1 Dominator coloring of lollipop graph

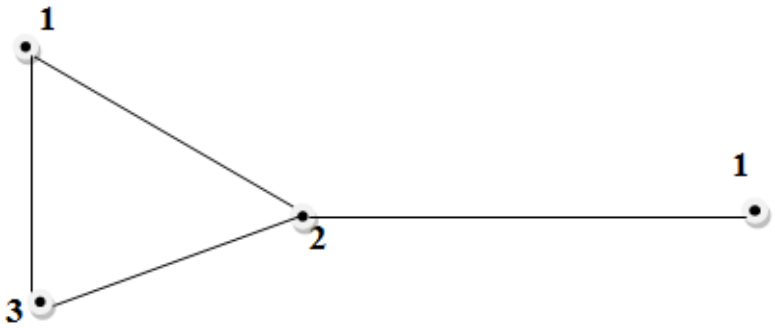


Fig 1 $(L_{3,1}) = 3$

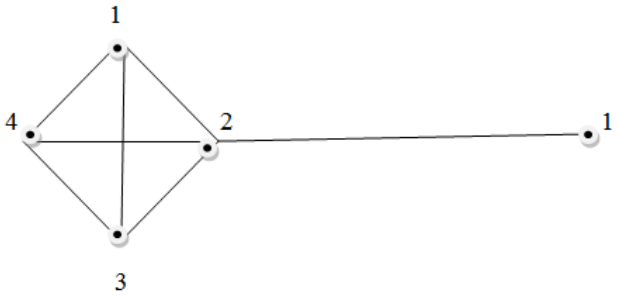


Fig 2 $(L_{4,1}) = 4$

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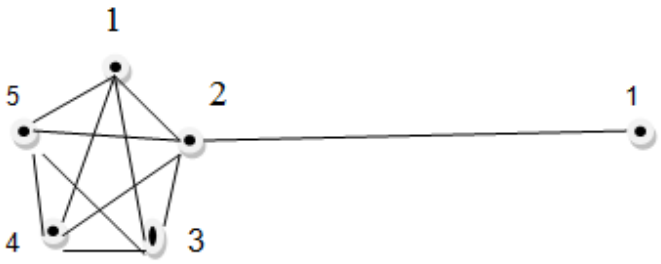


Fig 3 $(L_{5,1}) = 5$

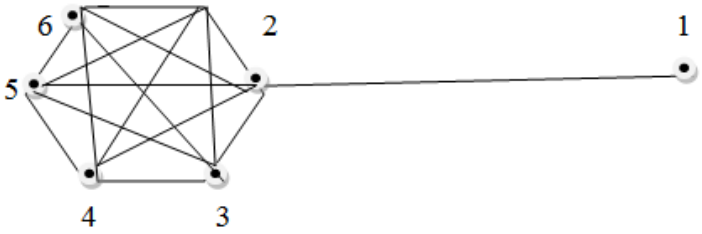


Fig 4 $(L_{6,1}) = 6$

Example 3.2

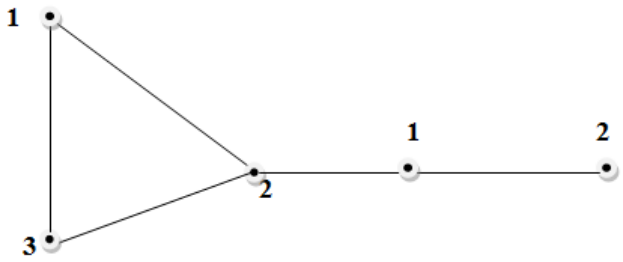


Fig 1 $(L_{3,2}) = 3$

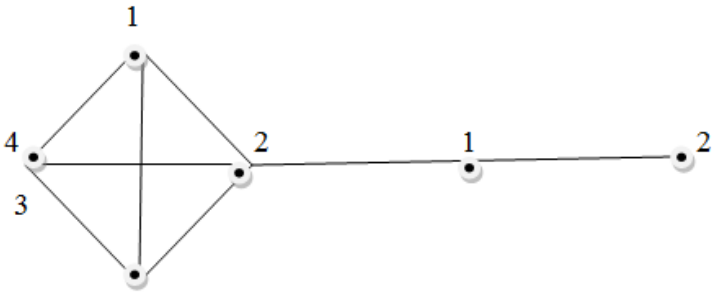


Fig 2 $(L_{4,2}) = 4$

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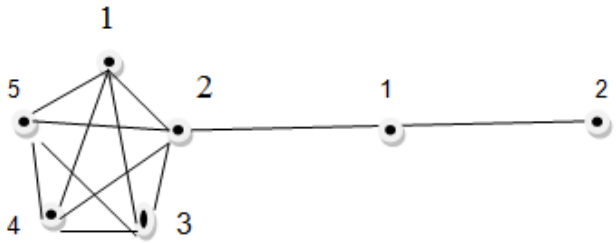


Fig 3 ($L_{5,2}$) = 5

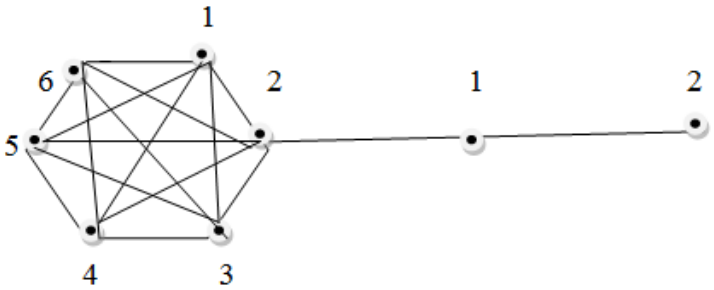


Fig 4 ($L_{6,2}$) = 6

CONCLUSION

In this paper we discussed that the dominator chromatic number of Lm,n .This paper can further be extended by identifying graph families of graphs for which these chromatic numbers are equal to other kinds of chromatic numbers.

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