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A New Class of Regular Space in Topological Spaces

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Abstract

The notion of this article is to introduce a new type of regular space namely gj-regular space in topological spaces. Characterization of the basic properties using these regular spaces have been discussed and studied.

Keywords: *j open set, j closed set, gj open set and gj regular space*

1. INTRODUCTION

Levine[8]initiated the study of generalized open sets with the introduction of semi open sets. S.N.Deeb, I.A.Hassanein and A.S.Mashour,[9]have defined a regular space in topological spaces. In general topology, pre open set plays an important role. The notions of pre – regularity and strong regularity is more important in generalization of regularity.

In 1982, A.S Manshhour et al [1] introduced the notion of pre –open sets and pre –open, pre- continuous mappings and has most important generalizations of regularity which appear as notions of pre-regularity along with the strong regularity . D.Sasikala and I.Arockiarani [4] initiated λα -J closed sets in generalized topological spaces The aim of our paper is to introduce and investigate the notions of gj – regular space.

2. PRELIMINARIES

Definitions 2.1: Let (X,τ) be a generalized topological space and A⊆X, then A is said to be -J-open if A ⊆int(pcl(A)). The complement of τ-J-open is called as τ-J-closed

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Definition 2.2: A subset B of a space (X,τ) is said to be generalized closed iff $cl(B) \subseteq U$ whenever $B \subseteq U$ and U is open in X. The complement of g- closed set is called g – open.

Definition 2.3: A topological space (X,τ) is said to be strongly regular if for each preclosed set B and each point, there exist disjoint preopen sets U and V in X such that $(B) \subseteq U$ and $x \in V$.

Definition 2.4: Let (X,τ) be a topological space , then it is said to be a g pre - regular space iff every g pre – regular closed set F and every point $x \notin F$,there exist disjoint pre- open sets U and V such that $F \subset U$ and $x \in V$.

Definition 2.5: A topological space (X,τ) is said to be regular if for any closed set F and any point $x \notin F$, there exist a neighbourhood U of x and neighbourhood V in F that are disjoint.

Definition 2.6: A topological space (X,τ) is said to be g – regular if for each g- closed set F and each point $x \notin F$, there exist disjoint open sets U and V in X such that $F \subseteq U$ and $x \in V$.

3. GJ – REGULAR SPACE

Definition 3.1: Let (X,τ) be a topological space , then it is said to be a gj- regular space iff every gj – regular closed set F and every point $x \notin F$,there exist disjoint j- open sets U and V such that $F \subset U$ and $x \in V$.

Definition 3.2: For each j – closed set B and each point $x \notin B$, there exist j – open sets U and V such that $B \subset U$ and $x \in V$ then the topological space (X,τ) is said to be strongly j – regular.

Definition 3.3 : Let (X,τ) be a topological space , if every gj- closed set in X is j closed then it is said to be j - $T_{1/2}$ space.

Lemma 3.4: A strongly regular j- $T_{1/2}$ space is gj – regular.

Proof: Let (X,τ) be a strongly regular space and j- $T_{1/2}$ space. Since (X,τ) is a j- $T_{1/2}$ space , then every gj- closed set in X is j-

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closed. ie., the class of gj- closed sets and j-closed sets coincide.Since (X,τ) be a strongly regular space provides that for each j- closed set B and each point $x \notin B$, there exist disjoint j – open sets U and V such that $B \subset U$ and $x \in V$.Therefore, for each gj- closed set B and each point $x \notin B$, there exist disjoint j – open sets U and V such that $B \subset U$ and $x \in V$, then (X,τ) is gj- regular.

Theorem3.5: A topological space (X,τ) is gj- regular iff every gj-closed set F and every point $x \notin F$, there exist j- open sets U and V such that $F \subset U$ and $x \in V$ and $jcl(U) \cap jcl(V) = \emptyset$.

Proof: If F is gj- closed set of a space (X,τ) and $x \notin F$. since (X,τ) is a gj- regular space , so there exist disjoint j- open sets U and V such that $F \subset U$ and $x \in V$ and

$U \cap V = \emptyset$

$\Rightarrow U \cap jcl(V) = \emptyset$ and $jcl(U) \cap V = \emptyset$

$\Rightarrow jcl(U) \cap jcl(V) = \emptyset$

Remark: The converse is not true.

Theorem 3.6: Let (X,τ) be a topological space, then the following statements are equivalent.

- (X,τ) is gj- regular space
- For each point $x \in X$, for each gj-open set W of x , there exist a j-open set V of x such that $jcl(V) \subseteq W$
- For each point $x \in X$, for each gj-closed set F not containing x , there exist a j-open set V of x such that $jcl(V) \cap F = \emptyset$

Proof:i $\Rightarrow$ ii**:**Let (X,τ) be a gj- regular space. Let W be a gj- open set containing a point $x \in X$.Since W^c is gj – closed set and $x \notin W^c$. hence by hypothesis there exist j-open sets U and V such that $W^c \subseteq U$, $x \in V$ and $U \cap V = \phi$

Now, $U \cap V = \phi$

$\Rightarrow V \subseteq U^c$

$\Rightarrow jcl(V) \subseteq jcl(U^c) = U^c$

Again $W^c \subseteq V \Rightarrow U^c \subseteq W$

By combining the above results, we get

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jcl (V) $\subseteq$ W

ii $\Rightarrow$ i :Let F be any gj- closed set and x $\notin$ F. Then x $\in$ F^c , F^c is gj – open set.

By hypothesis, there exist a j-open set V of x such that jcl(V) $\subseteq F^c$ Obviously , F $\subseteq$ (jcl(V))^c and (jcl(v))^c is a j- open set containing F and V $\cap$ (jcl(V))^c = $\emptyset$.

Therefore (X,τ) is gj – regular.

ii $\Rightarrow$ iii : Let x $\in$ X and F be a gj- closed set such that x $\notin$ F. Then x $\in$ F^c , where F^c is gj – open set.Now by hypothesis there exist a j-open set V of x such that

jcl(V) $\subseteq$ F^c

jcl (V) $\cap$ F = φ

iii $\Rightarrow$ iiLet x $\in$ X and W be a gj – open set containing x. since W^c is gj- closed set and x $\notin$ W^c.Hence by hypothesis, there exist a j – open set U containing x such that

jcl(U) $\cap$ W^c = φ

jcl(V) $\cap$ W^c = φ

jcl(U) $\subseteq$W

Hence the proof

Definition 3.7: A map f : X $\rightarrow$ Y is called

(1) M j – open : if f (V) is j – open set in Y for every j – open set V of X.

(2) j- irresolute : if f^{-1}(F) is j – open on X for every j – open set F in Y.

(3) gj- irresolute: if f^{-1}(F) is gj – closed in X for every gj – closed set F in Y.

Theorem 3.8: If (X,τ) is gj – regular space and f : (X,τ) $\rightarrow$ (Y,σ) is bijective , gj-irresolute and M j – open , then (Y,σ) is gj – regular.

Proof : Let y $\in$ Y and F be any gj- closed subset of (Y,σ) with y $\notin$ F. since f is gj-irresolute , f^{-1}(F) is gj- closed set in (X,τ). Since f is bijective. Let f(x) = y , then x $\notin$ f^{-1}(y). by hypothesis , there exist j- open sets U and V such that x $\in$ U and f^{-1}(F) $\subseteq$V. Since f is M j-open and bijective, so y $\in$ f(U) and F$\subseteq$f(V) ,

f(U) $\cap$ f(V) = f(U$\cap$ V) = φ.

Hence (Y,σ) is gj – regular space.

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Corollary 3.9: If f : $(X,^{\tau})$ → $(Y,^{\sigma})$ is gj-irresolute , M j closed and B is a gj – closed subset of $(X,^{\tau})$ then f(A) is gj- closed.

Theorem 3.10: If f : $(X,^{\tau})$ → $(Y,^{\sigma})$ is gj-irresolute , M j closed and injective and $(Y,^{\sigma})$ is gj- regular then $(X,^{\tau})$ is gj- regular.

Proof: Let F be any gj-closed set of $(X,^{\tau})$ and x ∉ F. since f is gj- irresolute , M j closed (by Corollary(3.8)). f(F) is gj- closed in Y and f(x) ∉ f(F).Since $(Y,^{\sigma})$ is gj – regular and so there exist disjoint j- open sets U and V in $(Y,^{\sigma})$ such that f(x) ∈ U and f(F) $\subseteq$ V. By hypothesis $f^{-1}(U)$ and $f^{-1}(V)$ ∈ JO (X) such that x ∈ $f^{-1}(U)$,

F $\subseteq$ $f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \varphi$.

Therefore $(X,^{\tau})$ is gj- regular.

Theorem 3.11: Let $(X,^{\tau})$ be a topological space then the following is equivalent:

- $(X,^{\tau})$ is gj – regular.
- Every gj- open set U is union ofgj-regular sets.
- Every gj- closed set A is an intersection of gj – regular sets.

Proof:(i) ⇒(ii):Let U be a gj- open set and let x ∈ U. If A=X-U, then A is gj – closed. Now assume that there exist disjoint open subsets W_1 and W_2 of X such that x ∈W_1 and A $\subseteq$ W_2. If V = pcl (W_1) , then V is j – closed and

$V \cap A \subseteq V \cap W_2 = \varphi$

Therefore x ∈V $\subseteq$U.Thus U is union of gj-regular sets.

(ii) ⇒(iii): this condition is obvious.

(iii) ⇒(i):Let B be gj- closed and let x ∉ B. by assumption , there exists a gj- regular set V such that B$\subseteq$V and x ∉ V. If U = X\V, then U is a j- open set containing x and U$\cap$ V = φ. .Thus $(X,^{\tau})$ is gj- regular.

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