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Numerical Integration: How the Error of the Trapezoidal and Simpson Rules Falls with the Number of Intervals

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ABSTRACT

Many integrals that arise in engineering cannot be evaluated by a formula, and are instead computed numerically, by dividing the region under the curve into strips and adding up their areas; how accurately this is done depends on how many strips are used and, on the rule, used to estimate each strip's area, and the two common rules, the trapezoidal and Simpson's, differ greatly in how fast their error falls as more strips are used. This study examines numerical integration and how the error of the trapezoidal and Simpson rules falls with the number of intervals. The error of each rule, in approximating a known integral, was examined as the number of intervals was increased, so that how fast the error fell, and how the two rules differed, could be seen. The error of both rules fell as the number of intervals grew, but at very different rates: the trapezoidal rule, which approximates each strip by a straight-topped trapezium, had an error that fell with the square of the number of intervals, so doubling the intervals cut the error to about a quarter; Simpson's rule, which fits a curve through each pair of strips, had an error that fell with the fourth power of the number of intervals, so doubling the intervals cut the error to about a sixteenth. Simpson's rule was therefore far more accurate than the trapezoidal for the same number of intervals, and its advantage widened rapidly as the intervals increased, reaching a given accuracy with far fewer intervals. There was a practical floor, however, set by rounding in the arithmetic, below which using more intervals ceased to help and could even harm. The higher-order rule is preferred because it buys accuracy far more cheaply, which matters when each evaluation of the function is costly. The study shows that numerical integration error falls with the number of intervals, faster for higher-order rules, and it discusses the implications for computing integrals in engineering.

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KEYWORDS: *Numerical integration, Quadrature, Trapezoidal rule, Simpson's rule, Order of accuracy, Error, Step size, Numerical methods, engineering mathematics*

INTRODUCTION

Integrals are everywhere in engineering, expressing areas, volumes, totals, averages, and the accumulated effect of varying quantities. Many of them, however, cannot be evaluated by any formula: the function has no antiderivative in closed form, or is known only as data. Such integrals must be computed numerically, by approximation. The basic idea is to divide the region under the curve into many narrow strips, estimate the area of each, and add them up; the more strips, and the better each estimate, the closer the result to the true integral (1, 6).

How the area of each strip is estimated defines the rule of integration. The simplest, the trapezoidal rule, treats the top of each strip as a straight line, approximating it by a trapezium. A better one, Simpson's rule, fits a curve through each pair of strips, following the function more faithfully. These rules differ in accuracy: for the same number of strips, one may be far closer to the true integral than the other (1, 2).

Two things govern the accuracy: the number of intervals, more strips giving a finer and so more accurate approximation, and the order of the rule, how fast its error falls as strips are added. This paper examines numerical integration and how the error of the trapezoidal and Simpson rules falls with the number of intervals, examining the error of each as the number of intervals is increased. The aim is to show how fast the error falls and how the two rules differ, and to discuss the implications for computing integrals in engineering.

LITERATURE REVIEW

Numerical integration, or quadrature, by dividing the interval and summing strip areas is a foundational subject of numerical analysis, the trapezoidal and Simpson rules being the basic members of the Newton-Cotes family, set out with their error formulae in all the texts (1, 2, 5). A central result is that the error of a rule falls with the number of intervals at a rate set by the order of the rule, faster for higher-order rules (2, 8).

QUADRATURE RULES AND THEIR ERROR

The literature describes the rules and their error:

- The trapezoidal rule approximates each strip by a trapezium; its error falls with the square of the intervals (1, 2).

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- Simpson's rule fits a curve through each pair of strips; its error falls with the fourth power (1, 2).
- A higher-order rule reaches a given accuracy with fewer intervals (5, 8).
- Rounding in the arithmetic sets a floor below which more intervals do not help (4).

A recurring theme is that error falls with the number of intervals, faster for higher-order rules.

The literature establishes the orders of the two rules: the trapezoidal rule's error falls in proportion to one over the square of the number of intervals, and Simpson's in proportion to one over the fourth power, so Simpson's is far more accurate for the same number of intervals and its advantage grows as the intervals increase (1, 2, 5). It treats the limit set by rounding error, which dominates once the approximation is very accurate so that further refinement ceases to help, and the broader family of higher-order and adaptive rules, throughout the texts on numerical analysis and engineering numerics (3, 4, 6, 7, 9, 10).

RESEARCH GAP

Although the error orders of these rules are foundational, a concise demonstration that shows how the error of the trapezoidal and Simpson rules falls with the number of intervals, bringing out their different rates, remains useful as a clear illustration. There is therefore value in a study that displays the error against the number of intervals and relates it to choosing a rule for computing integrals.

OBJECTIVES

The objectives of this study are:

- To examine the error of the trapezoidal and Simpson rules as the number of intervals is increased.
- To show how the trapezoidal error falls with the square of the number of intervals.
- To show how Simpson's error falls with the fourth power, far faster.
- To relate the difference, and the rounding floor, to computing integrals in engineering.

METHODOLOGY

1. Error Against Number Of Intervals

The error of each rule, in approximating a known integral, was examined as the number of intervals was increased, the trapezoidal rule approximating each strip by a trapezium and Simpson's fitting a curve through each pair of strips (1, 2). The elements are summarised in Table 1.

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Table 1: Elements of numerical integration

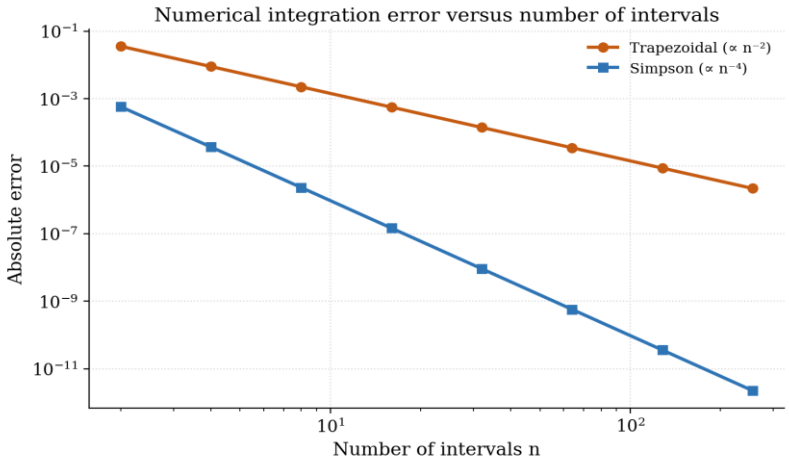
Element	Description
Intervals	Number of strips the region is divided into
Step size	Width of each strip
Trapezoidal rule	Each strip approximated by a trapezium
Simpson's rule	A curve fitted through each pair of strips

2. Comparing the Rules

The error of each rule was examined as the intervals increased, the trapezoidal error falling with the square and Simpson's with the fourth power of the number of intervals, so that their different rates could be seen (1, 2). The floor set by rounding, below which more intervals did not help, was noted (4).

RESULTS AND FINDINGS

The error of both rules fell as the number of intervals grew, but at very different rates, as shown in Figure 1, which plots the error of each rule against the number of intervals. The trapezoidal rule, approximating each strip by a straight-topped trapezium, had an error that fell with the square of the number of intervals, so doubling the intervals cut the error to about a quarter; Simpson's rule, fitting a curve through each pair of strips, had an error that fell with the fourth power, so doubling the intervals cut the error to about a sixteenth (1, 2).



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Figure 1: Numerical integration error versus number of intervals

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Simpson's rule was therefore far more accurate than the trapezoidal for the same number of intervals, and its advantage widened rapidly as the intervals increased, reaching a given accuracy with far fewer intervals (5). There was a practical floor, however, set by rounding in the arithmetic, below which using more intervals ceased to help and could even harm (4). The findings are summarised in Table 2.

Table 2: Error with number of intervals

Rule	Error falls as	Doubling intervals
Trapezoidal	1 / intervals^2	Error to about a quarter
Simpson's	1 / intervals^4	Error to about a sixteenth
Comparison	Simpson far more accurate	—
Very many intervals	Rounding floor	More does not help

DISCUSSION

The results show that numerical integration error falls with the number of intervals, faster for higher-order rules, and the difference in rate comes from how faithfully each rule follows the function. The trapezoidal rule replaces the function over each strip by a straight line; this misses the function's curvature, leaving an error in each strip, and the total error falls with the square of the number of intervals as the strips narrow. Simpson's rule fits a parabola through each pair of strips, capturing the curvature as well, so it matches the function much more closely; its error falls with the fourth power of the number of intervals. Following the function more faithfully thus yields a higher order, and a higher order means the error falls far faster as the intervals increase (1, 2).

This difference in order is decisive, and it is why higher-order rules are preferred. Because Simpson's error falls with the fourth power and the trapezoidal's only with the square, Simpson's is not merely better but better by a margin that grows rapidly as the intervals increase: each doubling of the intervals improves Simpson's result sixteenfold but the trapezoidal's only fourfold, so after a few doublings Simpson's is more accurate by an enormous factor. Equivalently, Simpson's reaches any required accuracy with far fewer intervals, hence far fewer evaluations of the function. This matters most when each evaluation is costly, as when the function comes from an expensive computation or experiment, for then the number

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of evaluations is the price of the integral, and the higher-order rule buys accuracy far more cheaply (2, 5).

The rounding floor is the important practical caveat, and it limits how far refinement can be pushed. The error analysed so far is the error of approximating the integral by strips, which falls as the intervals increase; but the arithmetic is done in finite precision, and each operation introduces a tiny rounding error. As the intervals are multiplied, the approximation error shrinks but the accumulated rounding error does not, and eventually the latter dominates, so that beyond some number of intervals the total error stops falling and may rise, refinement then doing more harm than good. There is thus an optimal number of intervals, beyond which more is wasteful; for a high-order rule like Simpson's this optimum is reached at relatively few intervals, since the approximation error is already tiny there, which is another reason to prefer it (4). For the engineer the lessons are to use a higher-order rule such as Simpson's to reach accuracy with few evaluations, to estimate the error so the result's accuracy is known, to stop refining once near the rounding floor, and, where the function varies unevenly, to use adaptive rules that place more intervals where the function is most curved (6, 7). The practical conclusion is that integrals are computed efficiently with higher-order rules whose error falls steeply with the intervals, down to the rounding floor, the main caveats being that order assumes a smooth function and that the figure illustrates the rates for one integral rather than all.

CONCLUSION

Numerical integration and how the error of the trapezoidal and Simpson rules falls with the number of intervals were examined by examining the error of each on a known integral as the intervals increased. The error of both fell as the intervals grew, but the trapezoidal rule's fell with the square of the number of intervals and Simpson's with the fourth power, so Simpson's was far more accurate for the same intervals and its advantage widened rapidly, reaching a given accuracy with far fewer intervals, until a floor set by rounding. The study shows that numerical integration error falls with the number of intervals, faster for higher-order rules, so that integrals are computed efficiently with higher-order rules such as Simpson's down to the rounding floor, with the caveats that the order assumes a smooth function and that the figure illustrates the rates for one integral rather than all.

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LIMITATIONS

The following limitations apply to this study:

- The orders of accuracy assume the integrand is smooth; sharp features reduce them.
- The figure illustrates the error rates for one integral rather than for all integrands.
- A floor set by rounding limits the accuracy attainable by refining the intervals.
- Only two fixed rules were compared; adaptive and higher-order rules were not examined.

FUTURE SCOPE

Future work could extend the present study as follows:

- Higher-order Newton-Cotes and Gaussian quadrature rules and their orders.
- Adaptive quadrature that concentrates intervals where the function is most curved.
- Behaviour of the rules on integrands with sharp features or singularities.
- Error estimation to choose the number of intervals automatically.
- The interplay of approximation and rounding error in determining the optimal intervals.

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