MANTECH Publications

Journal of Engineering Mathematics & Statistics
Volume 10, Issue 1, January-June 2026
ISSN: 3139-1222 (Online)

Taylor Series Approximation: How Adding Terms Extends the Range over Which a Polynomial Matches a Function

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ABSTRACT

Many functions in engineering are awkward to compute or analyse directly, and a common remedy is to approximate them near a point of interest by a polynomial, which is easy to evaluate; the Taylor series provides such polynomials, matching the function ever more closely as more terms are included, but only over a limited range around the point, which widens as terms are added. This study examines Taylor series approximation and how adding terms extends the range over which a polynomial matches a function. The Taylor polynomials of a function were examined as more terms were included, each polynomial compred with the function over a range around the point of expansion, so that how the match improves and its range widens with the number of terms could be seen. Each polynomial matched the function exactly at the point of expansion and well in a region around it, and adding terms both improved the match within that region and widened the region over which the match was good: a low-order polynomial followed the function only very near the point and departed from it quickly farther out, while higher-order polynomials hugged the function over a progressively wider range before eventually departing. The approximation was therefore best near the point and degraded with distance from it, the distance over which it remained good growing as terms were added. There was, however, a limit, set by the function, beyond which adding terms did not help, the range of good approximation not growing without bound for every function. The Taylor series is valued because it replaces an awkward function, near a point, by a simple polynomial of whatever accuracy is needed, which is why it underlies much numerical computation and analysis. The study shows that Taylor polynomials match a function near a point with a range that widens as terms are added, and it discusses the implications for approximation in engineering.

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KEYWORDS: *Taylor series, Polynomial approximation, Series expansion, Convergence, Remainder, Numerical computation, engineering mathematics, Order of approximation, Calculus*

INTRODUCTION

Functions such as the sine, the exponential, the logarithm, and many others arising in engineering cannot be evaluated by simple arithmetic, and may be awkward to manipulate in analysis. A widely used remedy is to replace such a function, near a point of interest, by a polynomial that closely matches it; polynomials involve only addition and multiplication, so they are easy to compute and to handle algebraically, and a good polynomial approximation lets the awkward function be treated as if it were a polynomial there (3, 7).

The Taylor series supplies these polynomials. Built from the value of the function and its derivatives at a chosen point, the Taylor polynomial of a given order matches the function and a number of its derivatives at that point, and so follows the function closely nearby. Including more terms, raising the order, makes the polynomial match the function more closely; the Taylor series is the limit of including all terms (1, 2).

But a Taylor polynomial matches the function well only over a limited range around the point of expansion, departing from it farther out, and this range, though it widens as more terms are added, does not always grow without bound. This paper examines Taylor series approximation and how adding terms extends the range over which a polynomial matches a function, examining the Taylor polynomials of a function as more terms are included. The aim is to show how the match improves and its range widens with the number of terms, and to discuss the implications for approximation in engineering.

LITERATURE REVIEW

The Taylor series and the theorem underlying it, that a sufficiently differentiable function can be approximated near a point by a polynomial in its derivatives there, with a remainder measuring the error, are foundational results of calculus and analysis, set out in the standard texts (1, 2, 6). The use of such series to approximate and compute functions is treated throughout the literature of engineering and physical mathematics (7, 8, 9).

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TAYLOR POLYNOMIALS AND THEIR ERROR

The literature describes the Taylor approximation and its limits:

- The Taylor polynomial matches the function and its derivatives at the point of expansion (1, 2).
- Adding terms improves the match and widens the range over which it is good (1, 3).
- The remainder measures the error and bounds how good the approximation is (2).
- The series converges only within a radius set by the function (10).

A recurring theme is that the approximation is best near the point and improves with more terms, within limits.

The literature establishes that the error of a Taylor polynomial, the remainder, is smallest at the point of expansion and grows with distance from it, and shrinks as more terms are added, so the approximation is best near the point and improves with order (1, 2). It treats the convergence of the full series, which holds only within a radius of convergence determined by the function, beyond which adding terms does not yield a good approximation, and the use of these results to approximate functions to required accuracy in computation and analysis, throughout the texts on calculus, analysis, and engineering mathematics (3, 4, 5, 7, 8, 10).

RESEARCH GAP

Although Taylor approximation is foundational, a concise demonstration that shows how successive Taylor polynomials hug a function over a widening range as terms are added remains useful as a clear illustration. There is therefore value in a study that displays a function with its Taylor polynomials of increasing order and relates the widening match to approximation in engineering.

OBJECTIVES

The objectives of this study are:

- To examine the Taylor polynomials of a function as more terms are included.
- To show how each polynomial matches the function near the point and departs farther out.
- To show how adding terms improves the match and widens its range.
- To relate the result, and its limits, to approximation in engineering.

METHODOLOGY

1. Taylor Polynomials of Increasing Order

The Taylor polynomials of a function were examined as more terms were included, each

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polynomial built from the function's value and derivatives at the point of expansion, and compared with the function over a range around that point (1, 2). The elements are summarised in Table 1.

Table 1: Elements of Taylor approximation

Element	Description
Point of expansion	Where the polynomial matches the function
Order	Number of terms included
Taylor polynomial	Polynomial approximating the function
Remainder	Error between polynomial and function

2. Adding Terms

The polynomials of increasing order were compared with the function over a range, so that the improvement of the match within the region and the widening of the region over which it was good could be seen (1, 3). The limit set by the radius of convergence, beyond which adding terms did not help, was noted (10).

RESULTS AND FINDINGS

Each polynomial matched the function exactly at the point of expansion and well in a region around it, and adding terms both improved the match and widened the region, as shown in Figure 1, which displays a function together with its Taylor polynomials of increasing order. A low-order polynomial followed the function only very near the point and departed from it quickly farther out, while higher-order polynomials hugged the function over a progressively wider range before eventually departing (1).

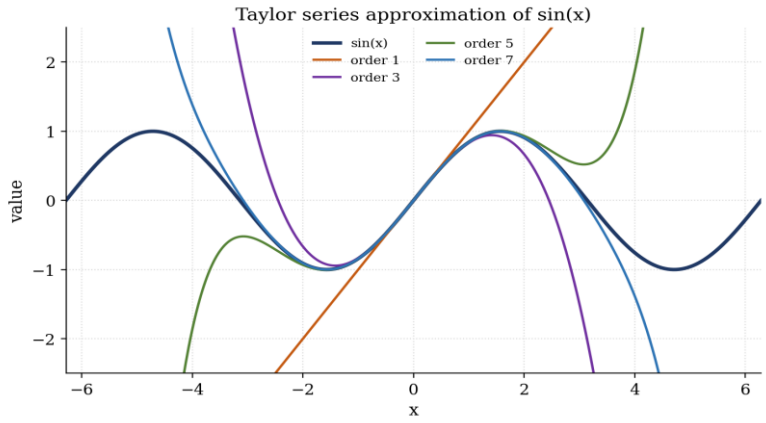


Figure 1: Taylor series approximation of sin(x)

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The approximation was therefore best near the point and degraded with distance from it, the distance over which it remained good growing as terms were added (2). There was, however, a limit, set by the function, beyond which adding terms did not help, the range of good approximation not growing without bound for every function (10). The findings are summarised in Table 2.

Table 2: Match with order of the Taylor polynomial

Order	Range of good match
Low	Only very near the point
Higher	A progressively wider range
At the point	Exact for every order
Beyond the radius of convergence	More terms do not help

DISCUSSION

The results show that Taylor polynomials match a function near a point with a range that widens as terms are added, and the behaviour follows from how the polynomial is constructed. The Taylor polynomial of a given order is built to agree with the function and its first several derivatives at the point of expansion; matching more derivatives forces the polynomial to bend like the function over a longer stretch, not merely to touch it. So the approximation is anchored at the point, where it is exact, and is good nearby; as the order rises and more derivatives are matched, the polynomial follows the function's curving over a wider neighbourhood before its own polynomial nature pulls it away. The error, accordingly, is least at the point and grows with distance, and shrinks at any fixed point as the order rises (1, 2).

This is why the approximation is local and why order and range are linked, which guides its use. Because the polynomial is tied to a single point, it is most trustworthy near that point; far from it, a low-order polynomial is useless, and even a high-order one eventually fails. To approximate a function over a range, then, one either uses enough terms that the polynomial stays accurate across that range, or expands about a point within the range so that the whole range lies near the centre; choosing the point of expansion close to where the function is to be evaluated keeps the order needed low. The order required for a given accuracy grows with the

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distance from the point, so the same accuracy is cheaper near the centre than far from it (3, 7).

The limit set by convergence is the important caveat, and it bounds what Taylor approximation can do. For some functions the series converges everywhere, so any range can be reached with enough terms; for others it converges only within a finite radius about the point, beyond which the polynomials diverge from the function no matter how many terms are added, because a singularity of the function limits the reach of the expansion. Within the radius, adding terms helps and the approximation can be made as accurate as desired; outside it, it cannot, and a different point of expansion or a different method is needed (2, 10). For the engineer, the practical lessons are therefore to expand about a point near where the function is needed, to take enough terms for the required accuracy over the range of interest, to estimate the error from the remainder so the accuracy is known, and to stay within the radius of convergence; within these, the Taylor series replaces an awkward function by a simple polynomial of whatever accuracy is wanted, which is why it underlies so much numerical computation and analysis, from evaluating functions to linearising equations (4, 7, 8). The practical conclusion is that Taylor approximation provides local polynomial substitutes for functions, accurate over a range that widens with order up to the radius of convergence, the main caveats being this radius and the growth of error with distance, and that the figure illustrates the behaviour for one function rather than all.

CONCLUSION

Taylor series approximation and how adding terms extends the range over which a polynomial matches a function were examined by examining the Taylor polynomials of a function as more terms were included. Each polynomial was exact at the point of expansion and good nearby, and adding terms improved the match and widened the range over which it was good, a low-order polynomial following the function only very near the point and higher orders over a progressively wider range, with a limit set by the radius of convergence beyond which more terms did not help. The study shows that Taylor polynomials match a function near a point with a range that widens as terms are added, so that the series provides local polynomial substitutes of whatever accuracy is needed, underlying much numerical computation and analysis, with the caveats of the radius of convergence and the growth of error with distance, and that the figure illustrates the behaviour for one function rather than all.

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LIMITATIONS

The following limitations apply to this study:

- The series converges only within a radius of convergence set by the function.
- The figure illustrates the behaviour for one function rather than for all functions.
- The error grows with distance from the point, so many terms are needed far from it.
- The function must be sufficiently differentiable at the point of expansion.

FUTURE SCOPE

Future work could extend the present study as follows:

- Error bounds from the remainder for required accuracy over a range.
- Choice of the point of expansion to minimise the order needed.
- Behaviour near singularities and the limits of the radius of convergence.
- Comparison of Taylor with other approximations such as Chebyshev and Padé.
- Use of Taylor expansions in linearising equations and in numerical methods.

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Received Date: May 23, 2026

Accepted Date: June 06, 2026

Published Date: June 12, 2026

Citation: Mukundan R. Seshadri, Brinda K. Parthasarathy, Hariprasad V. Ranganathan. Taylor Series Approximation: How Adding Terms Extends the Range over Which a Polynomial Matches a Function. Journal of Engineering Mathematics & Statistics. 2026; 10(1): 17-24p.

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