

## ***Noncooperative Gametheory --Cognitive Radio***

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### ***Abstract***

*Cognitive radio is an emerging technology that aims at utilizing the spectrum efficiently. Cognitive radio is an intelligent radio which has the ability to allocate the spectrum dynamically. Cognitive radio detects the unused spectrum holes (spectrum sensing), characterizes the available bands (spectrum decision), the coordination with other cognitive devices in the access phase (spectrum sharing), and the capability to handover towards other spectrum holes when licensed users if a better spectrum opportunity becomes available (spectrum mobility). In this paper, a game theoretic framework is proposed to evaluate Spectrum allocation functionalities in Cognitive Radio Networks. The secondary users are the players of the game and the spectrum selection process is done based on the game theoretic frame work. In this paper we proposed pure Nash and mixed Nash strategies and simulations are done in matlab. Numerical results are reported to assess the quality of the game equilibrium.*

***Keywords:*** *Cognitive Radio (CR), fairness, game theory, Nash Bargaining Solution*

### **I. INTRODUCTION**

Cognitive Radio Networks (CRNs) are emerging as a viable solution to solve spectrum shortage/efficiency problems. CRNs are based on cognitive devices

which are able to configure their transmission parameters (e.g., frequency band, waveforms, etc.) depending on the surrounding environment, consequently exploiting under-utilized spectrum

portions. The motivation for cognitive radio stems from various measurements of spectrum utilization which generally show that spectrum is under-utilized, in the sense that the typical duty cycle of spectrum usage at a fixed frequency and at a random geographical location is low. This means that there are many “holes” in the radio spectrum that may be exploited for use by wireless users other than the spectrum licensee. To this extent, even “unlicensed” (secondary) cognitive radio users can be allowed to use licensed spectrum, provided that they do not interfere with any primary licensed user. The achievable efficiency of CRNs depends on the specific cognitive capabilities (and functionalities) of the network devices. As a first step of the cognitive cycle, cognitive nodes must be able to detect unused (or under-utilized) spectrum portions (spectrum sensing), and to characterize them on the basis of several parameters (e.g., bandwidth, transmitting power), which can be collected through statistical information on primary activity (spectrum decision). Once spectrum holes have been identified and characterized, one of the most challenging issues concerns the exploitation of the available resources. Depending on the specific quality of service requirements, several functions can be adopted by users to select

the best available spectrum portion, and to coordinate with other cognitive devices in the access phase (spectrum sharing). Finally, secondary cognitive devices should support the capability to handover towards other spectrum holes when licensed users use a “better” spectrum opportunity becomes available (spectrum mobility). In this work, we address the CRN scenario where greedy and selfish secondary users opportunistically exploit the spectrum portions vacated by primary users. To this extent, we propose a non-cooperative game theoretic framework to study the inherent competition among secondary users in the cognitive spectrum selection process. Namely, the proposed framework accounts for: the time-varying radio environment in terms of availability and quality of the spectrum portions (spectrum decision); the interference among secondary users (spectrum sharing) the cost associated to spectrum handover (spectrum mobility). Since one of the most important aspects of cognitive terminals resides in their rationality, game theory may be conveniently used to provide formal tools to model the interactions and competitions among users, to derive equilibrium criteria and to study the optimality and the stability of the solution. We take here a constructive approach by analyzing, at first, a static game in which

spectrum mobility is neglected, and secondary users evaluate different spectrum opportunities, considering both the quality-of-service and the corresponding congestion level. Then, we move to a dynamic game formulation which accounts for the temporal evolution of the system, the corresponding time-varying primary user's activity, and the costs associated to spectrum mobility. In both cases, we characterize and derive the Nash equilibrium of the games, and we thoroughly comment on their quality.

## 2. GAME THEORY

Game theory – a mathematical model and technique developed in economics to analyze interactive decision processes, predict the outcomes of interactions, identify optimal strategies. Game theory techniques were adopted to solve many protocol design issues (e.g., resource allocation, power control, cooperation enforcement) in wireless networks. Fundamental component of game theory is the notion of a game. A game is described by a set of rational players, the strategies associated with the players, and the payoffs for the players.

Three major components in a strategic form game model are:

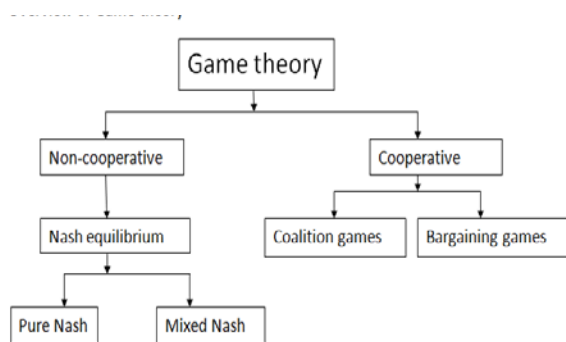
- A finite set of players - „N“
- A set of actions –  $A_k$ , for each player  $k$
- Payoff/utility function denoted by  $U_k$ , which measures the outcome for player  $k$

A game  $G$  is defined as

$$G = \{N, \{A_k\}_{k \in N}, \{U_k\}_{k \in N}\}$$

A rational player has his own interest, and therefore, will act by choosing an available strategy to achieve his interest. A player is assumed to be able to evaluate exactly or probabilistically the outcome or payoff (usually measured by the utility) of the game which depends not only on his action but also on other players. In Non-cooperative Game Theory Rational players having conflicting interests – e.g. scheduling in wireless networks. Defined by – Set of players – Set of strategies for each player. The players engage in the game while being selfish – Each player wishes to maximize his payoff or „utility“. Solution the Nash equilibrium – No user can unilaterally improve his payoff – Can be inefficient Static and Dynamic Games. In static games (one-shot games), the players make their moves in isolation without knowing what other players have

done. This does not necessarily mean that all decisions are made at the same time, but rather only as if the decisions were made at the same time. Dynamic games have a sequence to the order of play and players observe some, if not all, of one another's moves as the game progresses.



## GAME THEORY

### 1. Non-Cooperative Games:

- Matching game
- Cournot game
- Bertrand game
- Repeated game
- Super modular game
- Potential game
- Evolutionary game
- Auction game

### 2. Cooperative Games:

- Coalitional games
- Bargaining games

In non-cooperative game theory there are two ways in which a game can be

represented – normal form game (or strategic form game) and extensive form.

**Nash Equilibrium (NE):** A Nash equilibrium is an action profile  $a^*$  with the property that no player  $k$  can do better by choosing an action different from  $a_k^*$ , given that every other player  $j$  adheres to  $a_j^*$ . That is, for every player  $k$ ,  $u_k(a^*) \geq u_k(a_k, a_{-k}^*)$  for every action  $a_k$  of player  $k$ , where  $u_k$  is the payoff function for player  $k$ .

- Players: finite set of players,  $\{1, 2, \dots, N\}$
- Strategy Space: formed from the Cartesian product of each player's strategy set
- $A = A_1 \times A_2 \times \dots \times A_N$
- Payoffs: set of utility functions,  $\{u_1, u_2, \dots, u_N\}$

Nash equilibrium corresponds to a steady state of the game among "experienced players". It represents an outcome that results from the simultaneous maximization of individual payoffs. If a Nash equilibrium is common knowledge, then every player would indeed play the Nash equilibrium strategy, thereby

resulting in the Nash equilibrium being played.

In other words, a NE strategy profile is self-enforcing. If the players are searching for outcomes or solutions from which no player will have an incentive to deviate, then the only strategy profiles that satisfy such a requirement is the Nash equilibrium.

### 3. NON COOPERATIVE GAME THEORY

In game theory, a non-cooperative game is a game with competition between individual players and in which only self-enforcing) alliances (or competition between groups of players, called "coalitions") are possible due to the absence of external means to enforce cooperative behavior (e.g. contract law), as opposed to cooperative games

Non-cooperative games are generally analyzed through the framework of non-cooperative game theory, which tries to predict players' individual strategies and payoffs and to find Nash equilibria

- Nash Equilibrium is a term used in game theory to describe an equilibrium where each player's

strategy is optimal given the strategies of all other players.

- A Nash Equilibrium exists when there is no unilateral profitable deviation from any of the players involved. In other words, no player in the game would take a different action as long as every other player remains the same. Nash Equilibria are self-enforcing; when players are at a Nash Equilibrium they have no desire to move because they will be worse off.

**Necessary Conditions:** The following game doesn't have payoffs defined

	L	R
T	a , b	c , d
B	e , f	g , h

In order for (T,L) to be an equilibrium in dominant strategies (which is also a Nash Equilibrium), the following must be true:

- $a > e$
- $c > g$
- $b > d$
- $f > h$

In order for (T,L) to be a Nash Equilibrium, only the following must be true:

- $a \geq e$
- $b \geq d$

### RESULTS FOR PURE NASH:

```

Command Window
>> purenash

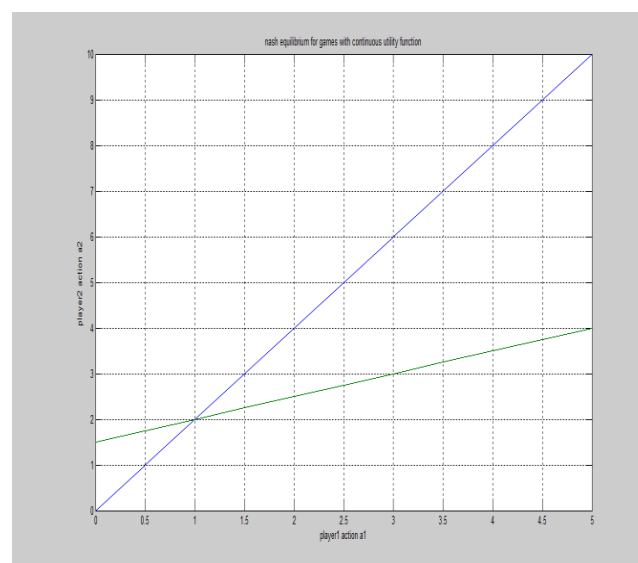
nash =

     4     7

nashi =

     1     2

fx >>
    
```



*Fig: computation of Nash equilibrium point for two players*

The saddle point (1,2) represents that the both players' first best responses are two and player one's second best response is one and the Nash utility or payoff is (4,7).

### MIXED STRATEGY NASH

- A mixed strategy of player i will be a collection of non-negative numbers which has unit sum and

are in one to one correspondence with his pure strategies.

- To write  $S_i = \sum \alpha_{i\alpha} \Pi_{i\alpha}$  with  $\sum \alpha_{i\alpha} = 1$  and  $\alpha_{i\alpha} \geq 0$  to represent such a mixed strategy, where the  $\Pi_{i\alpha}$ 's are the pure strategies of player i. We regard the  $S_i$ 's as points in a simplex whose vertices are the  $\Pi_{i\alpha}$ 's.
- This simplex may be regarded as a convex subset of a real vector space, giving us a natural process of linear combination for the mixed strategies.
- The suffixes i,j,k for players and  $\alpha, \beta, \gamma$  to indicate various pure strategies of a player. the symbols  $S_i, t_i, \gamma_i$ , etc will indicate mixed strategies;  $\pi_{i\alpha}$  will indicate ith players'  $\alpha$ th pure strategy, etc.
- In many games players choose unique actions from the set of available actions. These are called pure strategies. In some situations, though a player may want to randomize over several actions.

- If a player is choosing which action to play randomly, we say that the player is using a "mixed strategy" as opposed to a pure strategy.
- In a mixed strategy he chooses a probability distribution over the set of actions available to him.

Example:

strategy	L	R
U	(3,-2)	(2,3)
D	(2,2)	(5,0)

In the above example, the existence of pure nash point is not possible. Therefore we opt probabilistic idea of computing the nash point through mixed nash equilibrium concept.

### RESULT FOR MIXED NASH:

```

Command Window
>> mixedstrategynash
q =
    0.7500
q1 =
    0.2500
p =
    0.2057
p1 =
    0.7143
f %>>
    
```

The above results implies that for a two player game with two actions each, if player 1 plays strategy U with a probability 0.2857(p) then the player 2 plays strategy R with a probability of 0.25(1-q)(since  $\max(-2,3)$  is 3). Similarly if the player 1 plays strategy D with a probability of 0.7143(1-p) then the player 2 plays the strategy L with a probability of 0.75(q) (since  $\max(2,0)$  is 2).

### CONCLUDING REMARKS

Finally, we proved that game theory, a powerful tool to study the strategies of the players and their payoffs which can be applied in cognitive radio networks to study various strategies of the secondary users to access the available spectrum holes. we provided a comprehensive overview of game theory and its application to research on cognitive radio networks.

The concept of noncooperative game was explained along with the Nash equilibrium for mixed and pure strategies. Cognitive radio based on dynamic spectrum access is a new paradigm for designing wireless communications networks Efficient and robust algorithms need to be designed for dynamic spectrum access .

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