

Design and Manufacturing of a Gear Trains

L. Radhakrishna¹, N. Gopikrishna²

Assistant Professor^{1, 2}

Department of Mechanical Engineering

S R Engineering College, Warangal, Telangana, India

Corresponding Authors' Emails: *radha_krishna_l@srecwarangal.ac.in¹, gopikrishna-n@srecwarangal.ac.in²*

Abstract

Research is aimed at developing a mechanism using a gear train that can be used for lifting weights and loads. For the purpose of driving load we made necessary calculations for achieving end results. Paper reveals in designing relevant components for worthwhile works at higher expectations and estimations. A thought of that made us to prepare a prototype to lift required loads with lesser efforts. Paper sheaths Design calculations of various components stated and Manufacturing of those elements.

Keywords: *Gear Trains, Key, Shaft, Bearings, Drum, Rope, and other relevant Mechanical sytems and machines for preparation of a prototype*

1. INTRODUCTION

Mechanical advantaged mechanism was used in this mechanism. Mechanical advantage is a measure of an amplification related with force achieved by using a tool, which is a mechanical device or a machine system. Ideally, this machine system preserves the simply trades off forces against movement and input power to obtain a desired amplification.

Feature effect produced by Gear Train: Gear teeth is designed so that a number of teeth on a gear is relatively proportional to a radius of its pitch circle, and so that pitch circles of the meshing gears parts roll on each other without slipping. The speed ratio for a pair of meshing elements, gears can be computed from the ratio of the radii of the pitch circles and a ratio of the no: of teeth on each gear, its gear ratio.

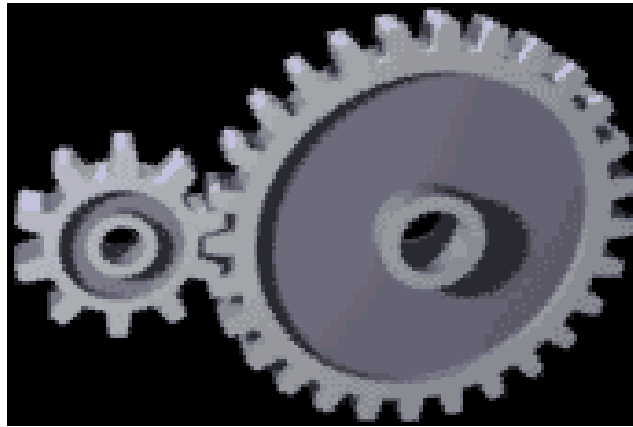


Figure 1: Two meshing gears transmit rotational motion

The velocity ‘v’ of a point of contact on these pitch circles is same on both the gears, and is given by

$$v = r_A \omega_A = r_B \omega_B,$$

Where, input gear A has radius r_A and meshes with output gear B of radius r_B , therefore,

$$\frac{\omega_A}{\omega_B} = \frac{r_B}{r_A} = \frac{N_B}{N_A}.$$

Here, N_A – no: of teeth on input gear and N_B – no: of teeth on output gear.

Mechanical advantage of a pair of meshing gears for which a input gear has N_A teeth & output gear has N_B teeth is given by

$$MA = \frac{T_B}{T_A} = \frac{N_B}{N_A}.$$

This shows that if an output gear G_B has more teeth than input gear G_A , then a gear train amplifies an input torque. if an output gear has fewer teeth than an input gear, then gear train reduces a input torque. If an output gear of a gear train rotates more slowly than the input gear, then the gear train is called as speed reducer. In this case, because the output gear must have more teeth than the input gear, the speed reducer will amplify the input torque.

In this work, we use this effect of mechanical machine advantage which is produced by the means of gear trains.

2. LITERATURE REVIEW:

J. Venkatesh et al., Designed and made an analysis of high speed helical gear [1].

Pinaknath et al., Designed and made an analysis of spur gear [2]. Sheng li et al., investigated the impact of design parameters on gear trains [3]. B. Venkatesh et al., Designed, Modeled and Manufactured of a gear [4].

3. EXPERIMENTATION:

3.1 Design Analysis:

Design of a Gear Train:

Let the velocity with which the weight is lifted be, $V = 4$ meters/min.

Diameter of the drum = 12 cm = 120 mm

Let the maximum load can be lifted by mechanism be 20 kilograms = $20 \times 9.81 = 196.2 \text{ N} = 200 \text{ N}$.

Calculations of a Gear shaft Diameter:

Design of Gear Shaft: The specification of performance requirements for shaft is given below in terms of power to be transmitted, and the speed of transmission: Performance specification required = 13.823 x 11 r.p.m.

Selection of material for part: Material selected is Mild Steel bright bar, whose mechanical properties are: Ultimate Tensile

Strength = 440 Mpa

Tensile Yield Strength = 370 Mpa

Hardness Number = 130

The material is a ductile material

Determination of External Loads Carried by Gear Shaft: Torque required transmitting power at the speed specified: The relationship between power and torque transmitted is given by the equation:

Power = $T \times \omega$ watts, Where, T = Torque transmitted in N-m ω = Angular velocity in radians/sec

Consequently, Power = $T \times \frac{2\pi N}{60}$ watts and $T = P \times \frac{60}{2\pi N}$

Where, N = Angular velocity of shaft in revs/min

Substituting for performance specification required

Torque to be transmitted is obtained

$T = 13.823 \times \frac{60}{2 \times \pi \times 11} = 12 \text{ N-m}$, From equation (I), the torque or twisting moment acting on the gear shaft is obtained as 15.756×10^3

Transverse load on gear shaft arising from the torque transmitted: Tangential force on gear teeth F required to transmit torque specified:

The torque transmitted is then given as function of the tangential tooth force and the pitch diameter of the gear as below: Torque

$$T = F \times \frac{d}{2} \text{ N-m}$$

Therefore, we get $F = T \times \frac{2}{d} \text{ N}$

Substituting for the pitch diameter of the input gear $d = 64 \text{ mm}$ and $T = 15.756 \times 10^3 \text{ N-mm}$

We get, $F = 15.756 \times 10^3 \times \frac{2}{64} = 492.375 \text{ N}$

Resultant force on gear tooth required to transmit torque specified: The tangential force F is the component of the resultant gear tooth force that gives rise to the transmitted torque. It acts at the pitch point, along the tangent to the pitch circle diameter of the gear tooth. The resultant gear tooth force is normal to the tooth surface, and therefore inclined to the tangent to pitch circle diameter at an angle equal to the pressure angle of the gear tooth.

The resultant force on the gear tooth is given by the equation, Resultant Force $R = \frac{F}{\cos \phi}$

where

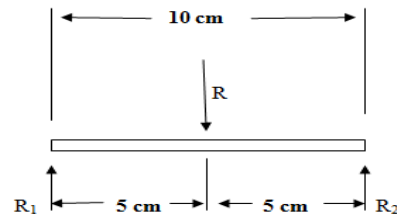
$\phi = \text{Pressure angle}$, $R = F \times \sec \phi$

Substituting for $F = 492.375 \text{ N}$ and $\phi = 20^\circ$

$$R = 492.375 \times \sec 20 = 1206.558 \text{ N}$$

Determine bending moment loads on the input shaft:

Loading diagram of the input shaft- considered as a simply supported beam subject to transverse point loads:



Weight of the Gear = $2.35 \text{ kg} = 2.35 \times 9.81 = 23.05$

$$R = 1206.558 + 23.05 \text{ N} = 1230.008 \text{ N}$$

Reactions at simple supports are,

$$R_1 = R_2 = \frac{R}{2} = \frac{1231}{2} = 615.5 \text{ N}$$

Shear force diagram: The direct shear stress induced by the shear force reaches its maximum value at the centre of the shaft, while the stresses caused by bending and

torsion reach their maximum values the surface of the shaft. the effect of the direct shear stress is therefore ignored.

Bending moment diagram for input shaft:

Diagram for a straight beam with intermediate load and simple supports is as shown in **figure 2** below

The maximum bending moment is then given by

$$M_{\max} = R \times \frac{l}{2} = 1231 \times \frac{100}{2} = 61550 \text{ N-mm}$$

External load on the gear shaft: The external load on the input gear shaft then reduces to combined torsion & bending, where the torsion and bending loads are:

$$T = 15.756 \times 10^3 \text{ N-mm}, M_{\max} = 61550 \text{ N-mm}$$

Determination of stresses induced by the external loads: Stresses due to combined torsion and bending of shaft. In this situation, there is a plane stress at the location of maximum bending moment as shown in **Figure 3**

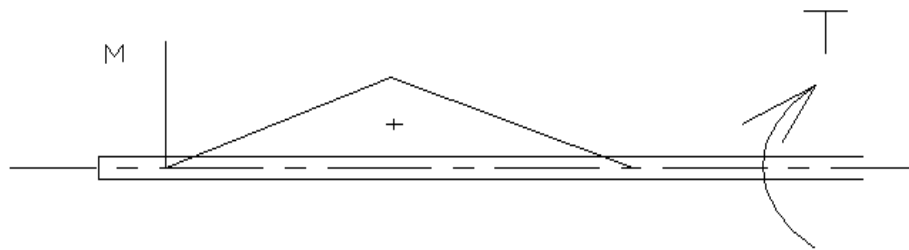


Figure 2: Diagram for a straight beam with intermediate load and simple supports

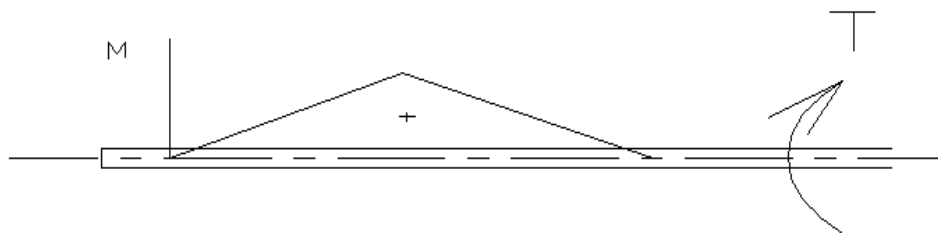


Figure 3: Maximum Bending Moment

The stress elements are

$$\sigma = \text{Normal stress due to bending} = \frac{32M}{\pi d^3}$$

$$\tau = \text{Stress due to torque} = \frac{16T}{\pi d^3}$$

Simplifying loading situation of input shaft into a static load which remains constant in spite of rotation of the shaft, to determine stress at location of highest stresses in terms of principal & maximum shear stress arising from the loads on the member

Applying maximum shear stress theory of failure

The Maximum Shear Stress theory of failure states:

When Yielding occurs in any material, the maximum shear stress at the point of failure equals or exceeds the maximum shear stress when yielding occurs in the tension test specimen.

Maximum shear stress in an element in terms of external loads: Substituting for σ and τ in equation for maximum shear stress yields the expression for maximum shear stress in terms of load and dimension of element as shown below

$$\tau_{\max} = \frac{1}{2} \sqrt{\sigma^2 + 4\tau^2} = \frac{1}{2} \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4 \left(\frac{16T}{\pi d^3}\right)^2}$$

$$\equiv \tau_{\max} = \frac{16}{\pi d^3} \sqrt{M^2 + T^2}$$

Shear strength of chosen (ductile) material:

The yield strength in shear of ductile materials such as steel is predicted to be half tensile yield strength by the maximum shear stress theory of failure, and the shear yield strength of such materials can therefore be derived from the tensile yield strength.

$$\text{Therefore } S_{sy} = \frac{S_y}{2}$$

Where, S_{sy} = Shear strength of the material

S_y = Yield strength of the material in tension

Compare significant stress with strength: design equation: Design equation then becomes

$$\tau_{\max} = \frac{16}{\pi d^3} \sqrt{M^2 + T^2} = \frac{S_{sy}}{f.s} = \frac{S_y}{2 \times f.o.s}$$

From above equation, we get

$$d^3 = \frac{16}{\pi} \sqrt{M^2 + T^2} \times \frac{2 \times f.o.s}{S_y}$$

Where, f.o.s = Factor of Safety, and

(1,1.2,1.6,2,2.5,3,4,5,6,8,10,12,16,20,25,30, 35,40,45,50,55,60,65,70,75,80,90,100 mm.)

$\frac{S_{sy}}{f.o.s} = \frac{S_y}{2 \times f.o.s} = \tau_d$, The design or allowable shear stress,

Taking nearest shaft of diameter 16 mm

The design equation then becomes

$$d^3 = \frac{16}{\pi} \sqrt{M^2 + T^2} \times \frac{2 \times f.o.s}{S_y}$$

Review design: Determine the actual factor of safety resulting from the use of the selected standard shaft size. Rewriting the design equation in terms of the factor of safety

$$f.o.s = \frac{S_{sy}}{\tau_{max}} = \frac{S_y}{2 \times \tau_{max}}$$

Solving design equation: Substituting the torque and bending moment loads into design equation

$$\text{But } \tau_{max} = \frac{16}{\pi} \sqrt{M^2 + T^2}$$

$T = 15.756 \times 10^3$ N-mm, $M_{max} = 61550$ N-mm

Substituting, $T = 15.756 \times 10^3$ N-mm

$$d^3 = \frac{32}{\pi} \sqrt{M^2 + T^2} \times \frac{f.o.s}{S_y}$$

$M_{max} = 61550$ N-mm And $d = 16$ mm

$$\tau_{max} = \frac{16}{\pi \times 30^3} \sqrt{61550^2 + 15756^2} = 79.038$$

Substituting for yield strength of chosen material and the factor of safety, Factor of safety = 2 and Tensile yield strength $S_y = 370$ Mpa

Mpa

Substituting $S_y = 370$ Mpa and $\tau_{max} = 79.038$ Mpa

$$d^3 = \frac{32}{\pi} \sqrt{61550^2 + 15756^2} \times \frac{2}{370}$$

$$f.o.s = \frac{S_y}{2} \times \frac{1}{\tau_{max}} = \frac{370}{2} \times \frac{1}{79.038} = 2.38$$

$d = 15.182$ mm, Select shaft size from preferred metric range, Select the shaft size to be used form the nearest size in the range of preferred metric sizes

Factor of safety = 2.3

For factor of safety 2.3 the diameter of the gear shaft is

$$d^3 = \frac{32}{\pi} \sqrt{61550^2 + 15756^2} \times \frac{2.38}{370} = 4164.916 \quad d = 16.089$$

Therefore the next standard diameter 20 mm must be taken as the diameter for the gear shaft.

Gear Shaft: As per the obtained dimension of 20 mm diameter, a shaft of bright bar type with diameter 20 mm is selected and is cut to the required lengths. A bright bar is a standard bar which does not require any further machining operations to be done on it and can be used directly.

Keyways: These are made on the gear shaft using a tool keyway cutter (image below in Figure 4). The cutter while rotating drills the shaft and when moved laterally, producing a key slot.

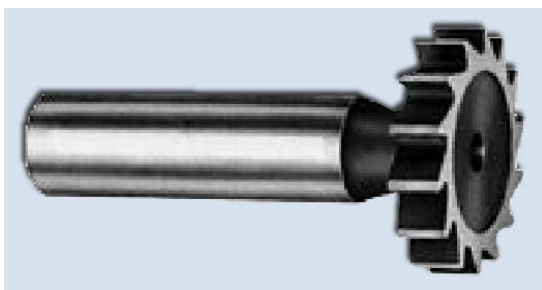


Figure 4: Keyways on Gear shafts

Key slots: These are the passages provided on the gear for the accommodation of key. A slotting machine is used to do this operation. It consists of a vertically moving tool, which during its reciprocating motion produces the slot in the gear.

Key: It is a small metal piece shaped to fit in a prepared key slot on the shaft. This is prepared using a metal piece. The metal piece is cut into the shape of the keyway and is grinded using a grinding machine until it is properly shaped and is fit into a slot and the keyway combination.

Frame: It is prepared using a mild steel rod of a L- Crossection and a Hollow square pipes. Hollow square rods, used as standing rods, supporting the frame and the supports provided for the bearings. The other supports for a frame are provided by L- cros sectioned rods. All rods are made into a frame by using an operation of welding.

Drum and Rope: Hollow pipe of 9 cm diameter made of G.I sheet, used for preparation of a drum. Two metal plates, Mild Steel are drilled with holes of dia 20 mm are made at centers of two plates. The gear shaft of 128 mm gear is inserted through plates and a drum, plates and shaft

are welded accordingly to prepare the required drum.

Before the welding operation, a hole is made at the center of the drum and the rope is inserted from the hole providing proper holding measures inside the drum.

Now, torque acting on the drum

$$= \frac{W \times 60 \times 10^{-3}}{2} = \frac{200 \times 60 \times 10^{-3}}{2} = 12 \text{ N-m}$$

Twisting moment, $M_t = 12 \text{ N-m} = \text{N-mm}$.

For speed of rotation of drum and gear D

Here, $V = 4 \text{ meters/min}$.

Using the equation

$$V = r\omega$$

(We know $r = 60 \times 10^{-3} \text{ meters}$)

$$\omega = \frac{4}{60 \times 10^{-3}} = 66.67 \text{ radians/min}$$

$$= 66.67 \times \frac{180}{\pi} \text{ degrees/min}$$

$$= 3819.71 \text{ degrees/min}$$

$$= \frac{3819.71}{360} \text{ revolutions/min (or) rpm}$$

$$= 10.61 \cong 11 \text{ rpm}$$

Therefore, rpm of drum (required) = 11 rpm
= rpm of Gear D.

Power to be transmitted from Gear D to Pinion C

$$P = \frac{2 \times \pi \times N \times T}{60} = \frac{2 \times \pi \times 11 \times 12}{60} = 13.823 \text{ watts}$$

Gear Ratio

We got speed of the gear D as $N_D = 11 \text{ rpm}$.

For speed reduction, Let us assume a gear ratio of 4.

We get, for the gear train ABCD,

$$\frac{N_A}{N_B} = \frac{N_A}{N_B} \times \frac{N_C}{N_D} = \frac{4}{1} = \frac{2}{1} \times \frac{2}{1}$$

[Since, $N_B = N_C$]

$$\text{Taking } \frac{N_A}{N_B} = \frac{2}{1} \text{ and } \frac{N_C}{N_D} = \frac{2}{1}$$

$$\text{From } \frac{N_A}{N_D} = \frac{4}{1}$$

We get $N_A = 4 \times N_D = 4 \times 11 = 44 \text{ rpm}$.

$$\text{From } \frac{N_A}{N_B} = \frac{2}{1}$$

We get $N_B = \frac{N_A}{2} = \frac{44}{2} = 22 \text{ rpm} = N_C$.

For Gear Ratio = 4,

We got $N_A = 44 \text{ rpm}$

$N_B = 22 \text{ rpm}$

$N_C = 22 \text{ rpm}$

$N_D = 11 \text{ rpm}$

Design Procedure:

For Gears C and D:

We have, Power to be transmitted, $P = 13.823 \text{ watts}$

Gear Ratio $i = 2$

Pinion Speed = 22 rpm

Gear Speed = 11 rpm

Table 1: Gear Material

Material	Ultimate Tensile Stress	Ultimate Yield Stress
Mild Steel	$\sigma_u = 440 \text{ N/mm}^2$	$\sigma_y = 370 \text{ N/mm}^2$

Calculation of Design Surface Stress and Bending Stress for the Selected Material:

$$[\sigma_c] = C_B \cdot HB \cdot K_{cl} \text{ (or)} [\sigma_c] = C_R \cdot HRC \cdot K_{cl}$$

For mild steel, Brinell hardness number (HB) is 130.

For HB-130 ≤ 350 , Coefficient $C_B = 25$ and $K_{cl} = 1$

For Life Cycles $\geq 10^7$

$$[\sigma_c] = 25 \times 130 \times 1 = 3250 \text{ kgf/cm}^2 = 325 \text{ N/mm}^2$$

$$[\sigma_b] = \frac{1.4 \times K_{bl} \times \sigma_e}{n K_\sigma} \quad \text{For Rotation in one direction only}$$

$$= \frac{K_{bl} \times \sigma_e}{n K_\sigma} \quad \text{For Rotation in both directions.}$$

N – Factor of safety - 2 (let)

$$K_\sigma = 1.4$$

$$K_{bl} = 1$$

For number of cycles greater than 10^7

$$\begin{aligned} \sigma_e &= \{0.22 (\sigma_u + \sigma_y) + 500\} \text{ Kg/sq.cm} \\ &= 0.22 (4400 + 3700) + 500 \\ &= 2282 = 228.2 \text{ N/mm}^2 \end{aligned}$$

$$[\sigma_b] = \frac{1.4 \times 1}{2 \times 1.4} \times 228.2 \text{ N/mm}^2 = 114.1 \text{ N/mm}^2$$

Therefore, Design surface Stress = 325 N/mm²

Design Bending Stress = 114.1 N/mm²

Minimum Centre Distance required for the Gear Drive: We have,

$$[M_T] = 12 \times 10^3 \times 1.3 = 15.6 \times 10^3 \text{ N-mm}$$

$$a \geq (i+1) \left\{ \left(\frac{0.74}{[\sigma_c]} \right)^2 \times \frac{E[M_T]}{i \phi} \right\}^{\frac{1}{3}}$$

$$= (2+1) \left\{ \left(\frac{0.74}{325} \right)^2 \times \frac{2.15 \times 10^5 \times 15.6 \times 10^3}{2 \times 0.3} \right\}^{\frac{1}{3}}$$

$$= 92.149 \cong 94 \text{ mm (let)}$$

Minimum Module:

$$m \geq 1.26 \left[\frac{[M_T]}{[\sigma_b] \phi_m Z_1 Y} \right]^{\frac{1}{3}}$$

$$m = 1.26 \left[\frac{15.6 \times 10^3}{114.1 \times 20 \times 10 \times 0.389} \right]^{\frac{1}{3}}$$

$$[Z_1 = 20 \text{ (Assume)}]$$

$$[\phi_m = \frac{b}{m} = 10 \text{ (Assume)}]$$

We get, $m = 1.52 = 2$ (take)

No. of teeth of pinion:

$$Z_1 = \frac{2a}{m(i+1)} = \frac{2 \times 94}{2 \times (2+1)} = 31.33 \cong 32 \text{ teeth}$$

$$\text{And } Z_2 = iZ_1 = 2 \times 32 = 64 \text{ teeth}$$

Pitch circle diameters:

$$d_c = d_1 = mZ_1 = 2 \times 32 = 64 \text{ mm}$$

$$d_D = d_2 = mZ_2 = 2 \times 64 = 128 \text{ mm}$$

Corrected Centre Distance:

$$a = \frac{d_1 + d_2}{2} = \frac{64 + 128}{2} = \frac{192}{2} = 96 \text{ mm.}$$

Face Width:

$$b = \phi \cdot a = 0.3 \times 96 = 28.2 \text{ mm} \cong 30 \text{ mm} -$$

Selected (Higher Value)

$$b = \phi_m \cdot m = 10 \times 2 = 20 \text{ mm}$$

Actual values of load concentration factor (K) and load factor (K_d) are found out as follows.

$$\phi_p = \frac{b}{d_1} = \frac{30}{64} = 0.468$$

Value of K – 1.01 ((Found out from the Table for the corresponding value of ϕ_p))

$$\text{Pitch line Velocity} = V = \frac{\pi d_1 N_1}{60 \times 1000} = \frac{\pi \times 64 \times 22}{60 \times 1000}$$

$$= 0.073 \text{ m/sec.}$$

For Cylindrical gear 8 and HB < 350, the value of K_d (Dynamic Load Factor) is equal to 1

Therefore,

$$[M_t] = M_t \times K \times K_d = 15.6 \times 10^3 \times 1.01 \times 1$$

$$= 15.756 \times 10^3 \text{ N-mm} - (I)$$

Checking of induced compressive stress and bending stress:

Induced surface compressive stress

$$\sigma_c = 0.74 \left\{ \frac{i+1}{a} \right\} \left\{ \frac{i+1}{ib} \times E[M_t] \right\}^{\frac{1}{2}}$$

$$= 0.74 \left\{ \frac{2+1}{96} \right\} \left\{ \frac{(2+1)}{2 \times 30} \times 2.15 \times 10^5 \times 15.756 \times 10^3 \right\}^{\frac{1}{2}}$$

$$= 300.96 \text{ N/mm}^2$$

(The above obtained value is less than the value of the material i.e., 325 N/mm². Hence, the design is safe)

Similarly, Finding out for bending stress,

$$\sigma_b = \frac{(i+1)[M_t]}{a.m.b.y} = \frac{(2+1)(15.756 \times 10^3)}{96 \times 2 \times 30 \times 0.358} = 22.922 \text{ N/mm}^2$$

(The obtained value is less than the value of the material i.e., 114.1 N/mm² Hence the design is safe)

Other parameters of the gear drive:

$$\text{Addendum, } h_a = f_o.m = (1 \times 2) = 2 \text{ mm}$$

$$\text{Dedendum, } h_f = (f_o + c) m = (1+0.25) \times 2 = 2.5 \text{ mm}$$

[$f_o = 1$ for full depth teeth] [c= 0.25 for full depth]

Tip circle diameter of pinion:

$$d_{a1} = (\text{pitch circle diameter of pinion}) + (2 \times \text{addendum})$$

$$d_{a1} = d_1 + 2h_a = 64 + (2 \times 2) = 68 \text{ mm}$$

Tip circle diameter of gear:

$$d_{a2} = (\text{pitch circle diameter of gear}) + (2 \times \text{addendum})$$

$$d_{a2} = d_2 + 2h_a = 128 + (2 \times 2) = 132 \text{ mm}$$

Root circle diameter of pinion:

$$d_{f1} = \text{Pitch circle diameter of pinion} - 2 \times (\text{Dedendum})$$

$$d_{f1} = d_1 - 2h_f = 64 - (2 \times 2.5) = 59 \text{ mm}$$

Root circle diameter of gear:

$$d_{f2} = \text{Pitch circle diameter of gear} - 2 \times (\text{Dedendum})$$

$$d_{f2} = d_2 - 2h_f = 128 - (2 \times 2.5) = 123 \text{ mm}$$

Tooth Height:

$$h = h_a + h_f = 2 + 2.5 = 4.5 \text{ mm.}$$

Table 2: For Gears C and D:

Gear Train	Proportion of machine cut teeth for module (m = 2)	
Addendum	m	2 mm
Dedendum	1.25m	2.5 mm
Tooth thickness	1.5708m	3.1416 mm
Tooth Space	1.5708m	3.1416 mm
Working Depth	2m	4 mm

Whole Depth	2.25m	4.5 mm
Clearance	0.25m	0.5 mm
Pitch Diameter	Z m	Gear = 128 mm, Pinion = 64 mm
Outside Diameter	(Z + 2) m	Gear = 132 mm, Pinion = 68 mm
Root Diameter	(Z - 2.5) m	Gear = 123 mm, Pinion = 59 mm
Fillet Radius	0.4 m	0.8 mm

Designing of Gears A and B:

Calculating the force and torque acting on the gear B: We Know, Torque acting on the gear D, For a weight of 20 kg. $T = 12 \times 10^3 \text{ N-mm}$

Taken force (for safety) for designing of gears is $T_D = 15.756 \times 10^3 \text{ N-mm}$

Diameter of Gear D = 128 mm

Gives, Radius = 64 mm

Therefore, Force acting on the teeth of gear D and Gear C (During Operation after mating) is obtained as,

$$F = \frac{T_D}{r} = \frac{15.756 \times 10^3}{64} = 246.32 \text{ N}$$

Diameter of Gear C = 64 mm

Radius = 32 mm

Therefore, Torque acting in the shaft, mounted by gears C and B is obtained a

$$T_C = T_B = 246.32 \times 32 = 7.882 \times 10^3$$

Gear Ratio, $i = 2$

Speed of pinion, $N_B = 44 \text{ rpm}$

Speed of Gear, $N_A = 22 \text{ rpm}$

Table 3: Gear Material

Material	Ultimate Tensile Stress	Ultimate Yield Stress
MildSteel	$\sigma_u = 440 \text{ N/mm}^2$	$\sigma_y = 370 \text{ N/mm}^2$

Design surface Compressive stress and Bending stress:

During the calculation of values for the gear train C and D, we got the design surface and bending stress values for mild steel. As same material is used here, same values can be used here.

The values are $[\sigma_c] = 325 \text{ N/mm}^2$

$[\sigma_b] = 114.1 \text{ N/mm}^2$

Minimum Centre Distance required for the Gear Drive:

We have,

$$\begin{aligned}
 [M_T] &= 7.882 \times 10^3 \text{ N-mm} \\
 a &\geq (i+1) \left\{ \left(\frac{0.74}{[\sigma_c]} \right)^2 \times \frac{E [M_T]}{i \phi} \right\}^{\frac{1}{3}} \\
 &= (2+1) \left\{ \left(\frac{0.74}{325} \right)^2 \times \frac{2.15 \times 10^5 \times 7.882 \times 10^3}{2 \times 0.3} \right\}^{\frac{1}{3}} \\
 &= 73.39 \cong 74 \text{ mm (let)}
 \end{aligned}$$

Minimum Module:

$$\begin{aligned}
 m &\geq 1.26 \left[\frac{[M_T]}{[\sigma_b] \phi_m Z_1 Y} \right]^{\frac{1}{3}} \\
 m &= 1.26 \left[\frac{7.882 \times 10^3}{114.1 \times 20 \times 10 \times 0.389} \right]^{\frac{1}{3}} \\
 [Z_1 &= 20 \text{ (Assume)}]
 \end{aligned}$$

$$[\phi_m = \frac{b}{m} = 10 \text{ (Assume)}]$$

We get, $m = 1.211 = 2$ (take)

No. of teeth of pinion:

$$Z_1 = \frac{2a}{m(i+1)} = \frac{2 \times 74}{2 \times (2+1)} = 24.667 \cong 25 \text{ teeth}$$

$$\text{And } Z_2 = iZ_1 = 2 \times 25 = 50 \text{ teeth}$$

Pitch circle diameters:

$$d_c = d_1 = mZ_1 = 2 \times 25 = 50 \text{ mm}$$

$$d_D = d_2 = mZ_2 = 2 \times 50 = 100 \text{ mm}$$

Corrected Centre Distance:

$$a = \frac{d_1 + d_2}{2} = \frac{50 + 100}{2} = \frac{150}{2} = 75 \text{ mm.}$$

Face Width:

$$b = \phi \cdot a = 0.3 \times 75 = 22.5 \text{ mm} \cong 23 \text{ mm} - \text{Selected (Higher Value)}$$

$$b = \phi_m \cdot m = 10 \times 2 = 20 \text{ mm}$$

Actual values of load concentration factor (K) and load factor (K_d) are found out as follows.

$$\phi_p = \frac{b}{d_1} = \frac{23}{50} = 0.46$$

Value of K – 1.01 ((Found out from the Table for the corresponding value of ϕ_p)

$$\text{Pitch line Velocity} = V = \frac{\pi d_1 N_1}{60 \times 1000} = \frac{\pi \times 64 \times 44}{60 \times 1000} = 0.115 \text{ m/sec.}$$

For Cylindrical gear 8 and HB < 350, approximately the value of K_d (Dynamic Load Factor) is equal to 1

Therefore,

$$[M_t] = M_t \times K \times K_d = 7.882 \times 10^3 \times 1.01 \times 1 = 7.96 \times 10^3 \text{ N-mm}$$

Checking of induced compressive stress and bending stress:

Induced surface compressive stress

$$\sigma_c = 0.74 \left\{ \frac{i+1}{a} \right\} \left\{ \frac{(i+1)}{i_b} \times E[M_t] \right\}^{\frac{1}{2}} = 0.74 \left\{ \frac{2+1}{96} \right\} \left\{ \frac{(2+1)}{2 \times 30} \times 2.15 \times 10^5 \times 7.882 \times 10^3 \right\}^{\frac{1}{2}} = 315.38 \text{ N/mm}^2$$

(The above obtained value is less than the value of the material i.e., 325 N/mm². Hence, the design is safe)

Similarly, Finding out for bending stress,

$$\sigma_b = \frac{(i+1)[M_t]}{a.m.b.y} = \frac{(2+1)(7.882 \times 10^3)}{74 \times 2 \times 24 \times 0.764} = 8.718 \text{ N/mm}^2$$

(The obtained value is less than the value of the material i.e., 114.1 N/mm² Hence the design is safe)

Other parameters of the gear drive:

$$\text{Addendum, } h_a = f_o.m = (1 \times 2) = 2 \text{ mm}$$

$$\text{Dedendum, } h_f = (f_o + c) m = (1+0.25) \times 2 = 2.5 \text{ mm}$$

[$f_o = 1$ for full depth teeth] [$c = 0.25$ for full depth]

Tip circle diameter of pinion:

$$d_{a1} = (\text{pitch circle diameter of pinion}) + (2 \times \text{addendum})$$

$$d_{a1} = d_1 + 2h_a = 50 + (2 \times 2) = 54 \text{ mm}$$

Tip circle diameter of gear:

$$d_{a2} = (\text{pitch circle diameter of gear}) + (2 \times \text{addendum})$$

$$d_{a2} = d_2 + 2h_a = 100 + (2 \times 2) = 104 \text{ mm}$$

Root circle diameter of pinion:

$$d_{f1} = \text{Pitch circle diameter of pinion} - 2 \times (\text{Dedendum})$$

$$d_{f1} = d_1 - 2h_f = 50 - (2 \times 2.5) = 45 \text{ mm}$$

Root circle diameter of gear:

$$d_{f2} = \text{Pitch circle diameter of gear} - 2 \times (\text{Dedendum})$$

$$d_{f2} = d_2 - 2h_f = 100 - (2 \times 2.5) = 95 \text{ mm}$$

Tooth Height:

$$h = h_a + h_f = 2 + 2.5 = 4.5 \text{ mm.}$$

Table 4: For Gears A and B

Gear Term	Proportion of machine cut teeth for module (m = 2)	
	M	
Addendum	M	2 mm
Dedendum	1.25m	2.5 mm
Tooth Thickness	1.5708m	3.1416 mm
Tooth Space	1.5708m	3.1416 mm
Working Depth	2m	4 mm
Whole Depth	2.25m	4.5 mm
Clearance	0.25m	0.5 mm
Pitch Diameter	Z m	Gear = 100 mm Pinion = 50 mm
Outside Diameter	(Z + 2) m	Gear = 104 mm Pinion = 54 mm
Root Diameter	(Z - 2.5) m	Gear = 95 mm Pinion = 45 mm
Fillet Radius	0.4 m	0.8 mm

Fundamental Law of Gear-Tooth Action:

Figure shows two mating gear teeth, in which

Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point K.

N_1N_2 is the common normal of the two profiles.

N_1 is the foot of the perpendicular from O_1 to N_1N_2

N_2 is the foot of the perpendicular from O_2 to N_1N_2 .

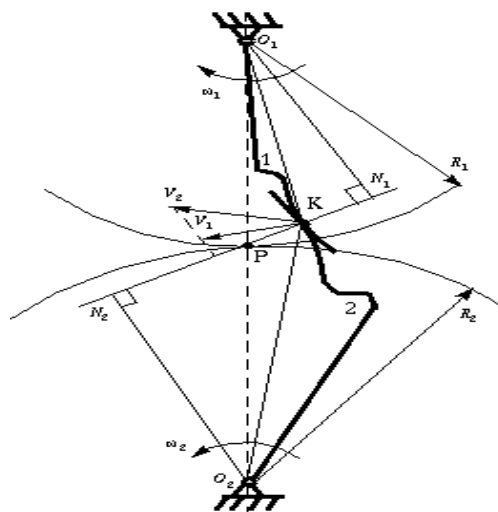


Figure 2: Two gearing tooth profiles

Although the two profiles have different velocities V_1 and V_2 at point K, their velocities along N_1N_2 are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other. Therefore, we have

$$O_1N_1 \cdot \omega_1 = O_2N_2 \cdot \omega_2$$

or

$$\frac{\omega_1}{\omega_2} = \frac{O_2N_2}{O_1N_1}$$

We notice that the intersection of the tangency N_1N_2 and the line of center O_1O_2 is point P, and

$$\Delta O_1N_1P \sim \Delta O_2N_2P$$

Thus, the relationship between the angular velocities of the driving gear to the driven gear, or **velocity ratio**, of a pair of mating teeth is

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

Point P is very important to the velocity ratio, and it is called the **pitch point**. Pitch point divides the line between the line of centres and its position decides the velocity ratio of the two teeth. The above expression

is the **fundamental law of gear-tooth action**.

Checking of Gear Train for the satisfaction of Law of Gearing:

For Gears C and D,

Corrected centre distance,

$$a = \frac{d_1 + d_2}{2} = \frac{64 + 128}{2} = 96 \text{ mm}$$

Diameters of Gears C and D,

$$D_C = d_1 = mZ_1 = 2 \times 32 = 64$$

$$D_D = d_2 = mZ_2 = 2 \times 64 = 128$$

The Radii of Gears

$$R_C = 32 \text{ mm}$$

$$R_D = 64 \text{ mm}$$

To satisfy the gearing law, the below equation should be satisfied by the gears,

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

O_1P, O_2P are the distances from the centre of the gears to the pitch circles.

ω_1, ω_2 are the angular velocities of the gears.

For Gears C and D,

To satisfy it we have to get,

$$\frac{\omega_1}{\omega_2} = \frac{R_D}{R_C}$$

Speeds of the gears C and D

$$N_C = 22 \text{ rpm}, N_D = 11 \text{ rpm}$$

$$\frac{\omega_C}{\omega_D} = \frac{\frac{2\pi N_C}{60}}{\frac{2\pi N_D}{60}} = \frac{N_C}{N_D} = \frac{22}{11} = 2 \text{ - (equation 1)}$$

Now,

$$\frac{R_D}{R_C} = \frac{64}{32} = 2 \text{ - (equation 2)}$$

From the above two equations we can say that the gears C and D satisfy the law of gearing.

Similarly, For the Gears A and B,

$$N_A = 44 \text{ rpm}, N_B = 22 \text{ rpm}$$

$$\frac{\omega_A}{\omega_B} = \frac{\frac{2\pi N_A}{60}}{\frac{2\pi N_B}{60}} = \frac{N_A}{N_B} = \frac{44}{22} = 2 \text{ - (equation 3)}$$

$$\frac{R_B}{R_A} = \frac{50}{25} = 2 \text{ - (equation 4)}$$

Therefore, from the above two equations we can say that gears A and B also satisfy the law of gearing.

CONCLUSIONS

In the present paper we have produced a gear train and tested it with finite loads such results explained in detail Thus, It is

concluded from this journal that there is a scope of using gears to lift the loads with lesser efforts.

REFERENCES

- [1] J. Venkatesh, int. Journal of engineering research and applications, issn : 2248-9622, vol. 4, issue 3(version 2), march 2014, pp.01-05
- [2] pinaknath. D, international journal of mechanical engineering and technology (ijmet)volume 7, issue 5, september–october 2016, pp.209–220,
- [3] Sheng li, journal of advanced mechanical design, systems and manufacturing
- [4] K. Miloš, i. Jurić, p. Škorput: aluminium-based composite materials in construction of transport means , promet – traffic&transportation, vol. 3, 2009, no. 2, 146-158