

Analysis of Nonlinear Large Deflection Observed in Stiffened Plates

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Abstract

This paper presents a report on the analysis of stiffened plates, that are basic structural members in marine structures, and include also aeronautic and space shuttles among other structures. Due to the simple fabric with a very high strength-to-weight ratio, these plates are also commonly used for building land based structures which include box girder and plate girder bridges. The stiffened plate has a number of one-sided stiffeners in either one or both directions, the latter configuration being also called a grillage.

Keywords: *Stiffened Plates, Tanaka & Endos' Experiments, Ghavami's Experiments*

INTRODUCTION

Stiffened plates are basic structural members in marine structures as shown in Figure 1, and include also aeronautic and space shuttles among other structures. Due to the simplicity in their fabrication and high strength-to-weight ratio, stiffened plates are also widely used for construction of land based structures such as box girder and plate girder bridges. The stiffened plate has a number of one-sided stiffeners in either one or both directions, the latter configuration being also called a

grillage (Figure 2). Ultimate limit state design of Stiffened plates' structures requires accurate knowledge about their behaviour when subjected to extreme loading conditions.

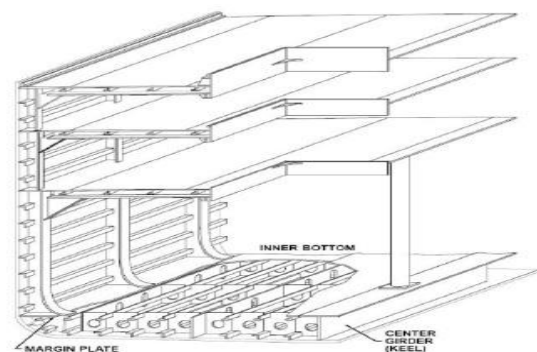


Figure 1: Some examples of thin-walled structures

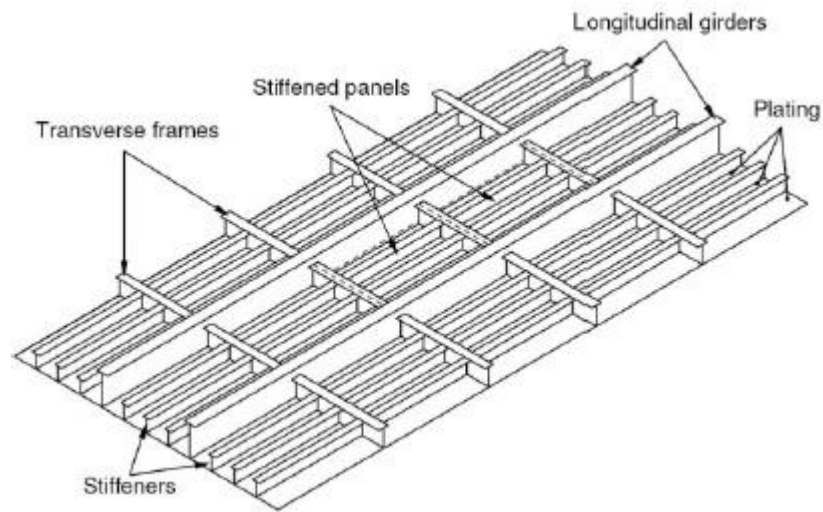


Figure 2: Structure of stiffened plates of the grillage type

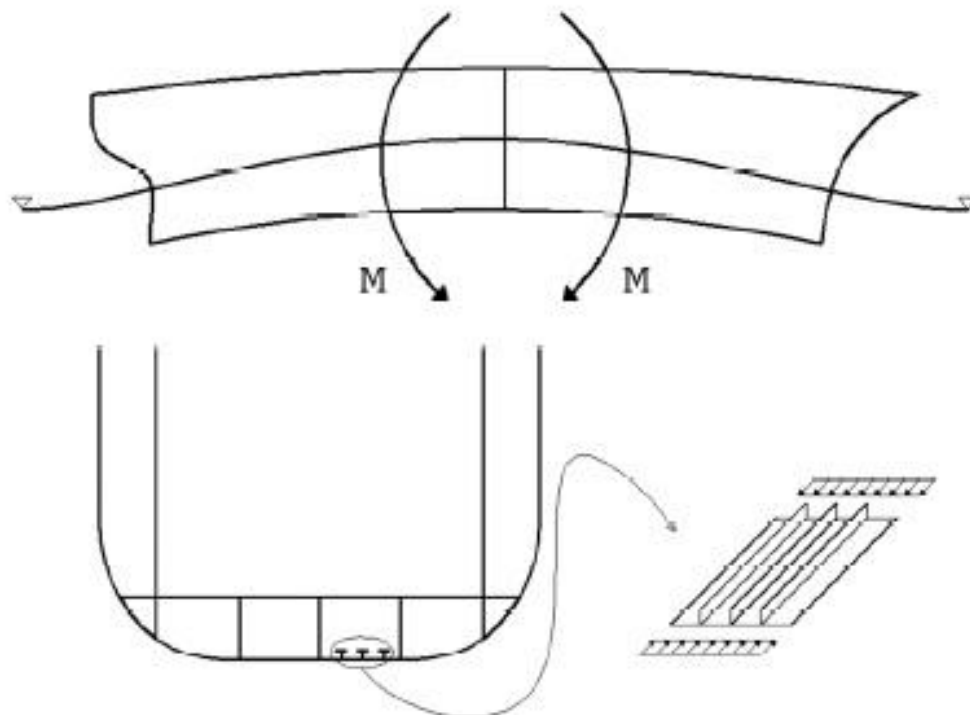


Figure 3: In-plane loading of stiffened plates when longitudinal bending of ship hull girder

One of the most important loads applied on stiffened plates is the longitudinal in plane axial compression arising for instance from longitudinal bending of the ship hull girder as presented in Figure 3. The need to improve our knowledge of the buckling modes of such plates was emphasised after the collapse of several offshore structures and some ships in Brazil as well as the failure of several box girder bridges in the seventies of the twentieth century, Merrison Committee [1], Crisfield [2], Murray [3], Frieze, et.al. [4]. Stiffened plates are efficient structures, as a large increment of the strength is created by a small addition of weight in the form of stiffeners. However the collapse mechanisms of stiffened plates under predominantly compressive load present a complex engineering problem due to the large number of possible combinations of plate and stiffener geometry, materials, boundary conditions and loading. The design of such structure has to meet several requirements such as minimization of the weight and maximization of the buckling load. Thus, the designer of this structure is confronted with the problem of satisfying two conflicting objectives; such problems are called multi-objective or vector optimisation problems. In general, the objective-functions do not attain their

optimum in a common point of the feasible points, Brosowski & Ghavami [5, 6].

For the analysis of such structural elements, the theory of orthotropic plate can be used to predict the global buckling stresses but not the local buckling and the interaction between the plate and the stiffeners, for the predominantly in-plane loading. In stiffened plates the initial imperfections due to the fabrication are inevitable. The buckling mechanism of stiffened plates depends, strongly, on the direction of initial bows, i.e. whether they are towards the plate or the stiffener. In the former case, the collapse is sudden due to buckling of the stiffener in contrast to the latter case, where a gradual failure occurs. Despite a substantial amount of theoretical research into the ultimate load behaviour of stiffened plates subjected to predominately in-plane loading, the accuracy and reliability of the predicted collapse load considering all the variables is not yet well confirmed. Specifically, in the available literature, no systematic theoretical and experimental investigation of the geometrical shape of the stiffeners cross-section on the ultimate buckling load behaviour of the stiffened plates, the interaction between the stiffeners and the

plate, which is the objectives of this paper is being presented.

The buckling behaviour of stiffened plates under different loading conditions which has been the topic of the authors investigation, both experimentally and numerically, during last three decades has been reviewed concisely in this chapter. Chen et al. [7] carried out experimental investigations on 12 stiffened plates under in-plane longitudinal compression, purely or in combination with lateral load. The specimens were in different damage conditions: seven “as-built”, two “dented” and three “corroded”. Hu and Jiang [8] simulated some of the tests made by Chen et al. [7], using the commercial program ADINA [9] and in-house program VAST [10], both based on the FEM. The former was used to analyse the “as-built” and “dented” stiffened plates, whereas the “corroded” specimens were analysed using VAST [4]. It was found, that in most cases the FEM produced similar responses to those of experimental results up to the loss of structural continuity. Grondin et al. [11] made a parametric study on the buckling behaviour of stiffened plates using the FEM-based commercial program ABAQUS [12]. Sheikh et al. [13] extended the studies in [11] to investigate the combined effect of in-plane

compression and bending using the same program. In these studies, only tee-shape stiffeners, plate aspect ratios, plate-to-stiffener cross-sectional area ratio with different initial imperfections of the plates were investigated.

All the cited studies, either experimentally or numerically, investigated the strength behaviour of longitudinally stiffened plates with specific boundary conditions. The continuity of both plates and stiffeners in thin-walled structures, composed of stiffened plates, leads to an interaction among the adjacent panels. Among the several available experimental investigations, two series of well executed experimental data on longitudinally multi-stiffened steel plates, with and without transversal stiffeners subjected to uniform axial in-plane load carried out to study the buckling and post-buckling up to final failure have been chosen. The first series are those of Ghavami [14] where the influences of stiffener cross-section of the type rectangular (R), L and T, as shown in Figure 4, have been investigated. The spacing of the stiffeners and the presence of rigid transversal stiffeners on the buckling behaviour up to collapse have also been studied. The second series of Tanaka & Endo [15], where the behaviour of stiffened plates have three and two flat

bars for longitudinal and transversal stiffeners respectively, were analysed. Besides, owing to the recent progress in the field of finite element method and available powerful FEM programs, it has been possible to assess the structural behaviour of the considered plates and stiffeners subjected to any combination of loads.

technique is employed to trace a full-range of elastic-plastic behaviour of the stiffened plates. It is seen that the FEM-based software is capable and accurate enough to simulate the test results. With the availability of high memory and high speed PCs', FEM programs become fast and cheap means to predict the buckling

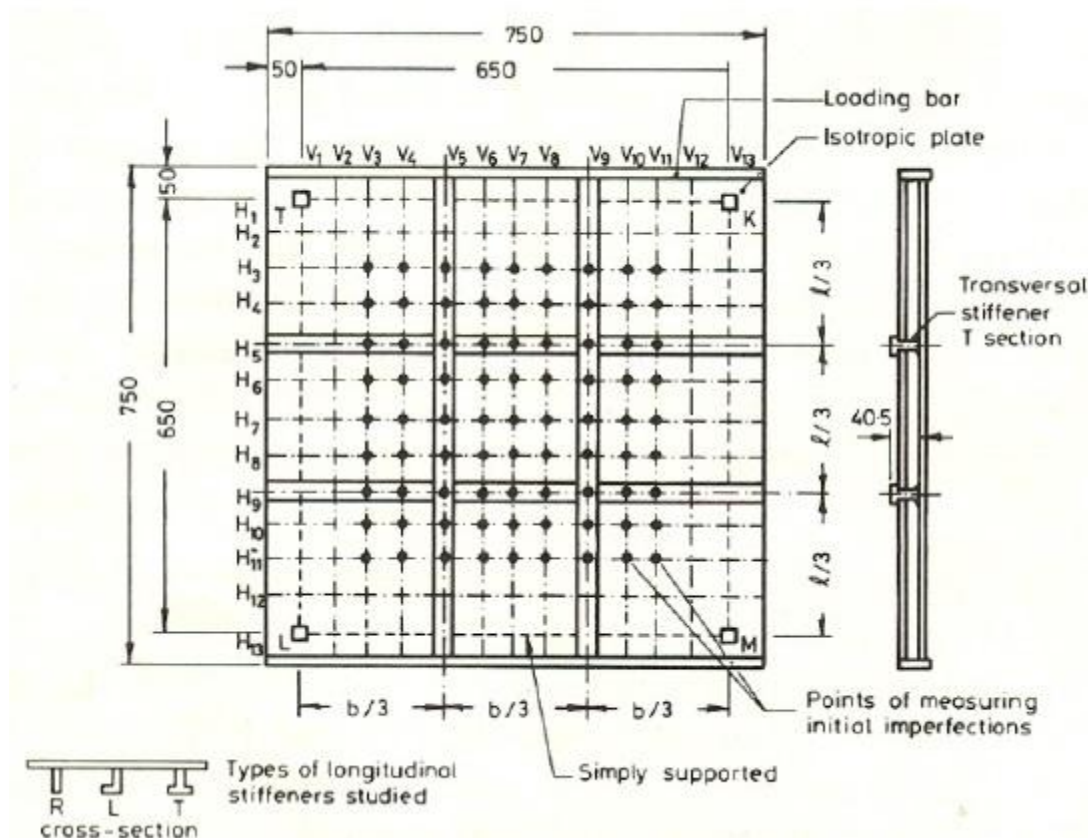


Figure 4: Ghavami's Test Models

Therefore one of the principal aims of this chapter is to present the applicability of the finite element method to simulate test results. The Finite Element Method (FEM)

and post-buckling behaviour of stiffened plates with different configurations up to collapse. Successful simulations using FEM-based software means, that plate

with different dimensions under various types of loading combinations and damages can be studied numerically. Besides, validated simulations using such programs enhance estimation of the ultimate strength analysis of box-like thin-walled structures composed of plates and stiffened plates.

GHAVAMI'S EXPERIMENTS

Ghavami [14] tested a total number of 17 plate models of overall dimensions $B=L=750$ mm in a specially designed testing rig as shown in Figure 5. The models were divided into six series, with their definition and dimensions summarised in Tables 1 and 2.

The average thickness of the plate was $t=4.4$ mm for the longitudinally stiffened plates with one and two rectangular (R), L and T stiffeners, designated as P1R, P1L, P1T and P2R, P2L, P2T respectively. The thickness of the plates, stiffened longitudinally, as for the series II and III but with one or two transversal stiffeners of T sections, P1R1T, P1L1T, P1T1T, P2R1T, P2L1T, P2T1T and P2R2T, P2L2T, P2T2T respectively was equal to 4.8mm. The span between the simple supports for all models was 650mm in both directions. In each group one isotropic plate, P1, P2 was also tested as a reference model. However the supports for the longitudinally stiffened plates, series II and III were not continuously simply

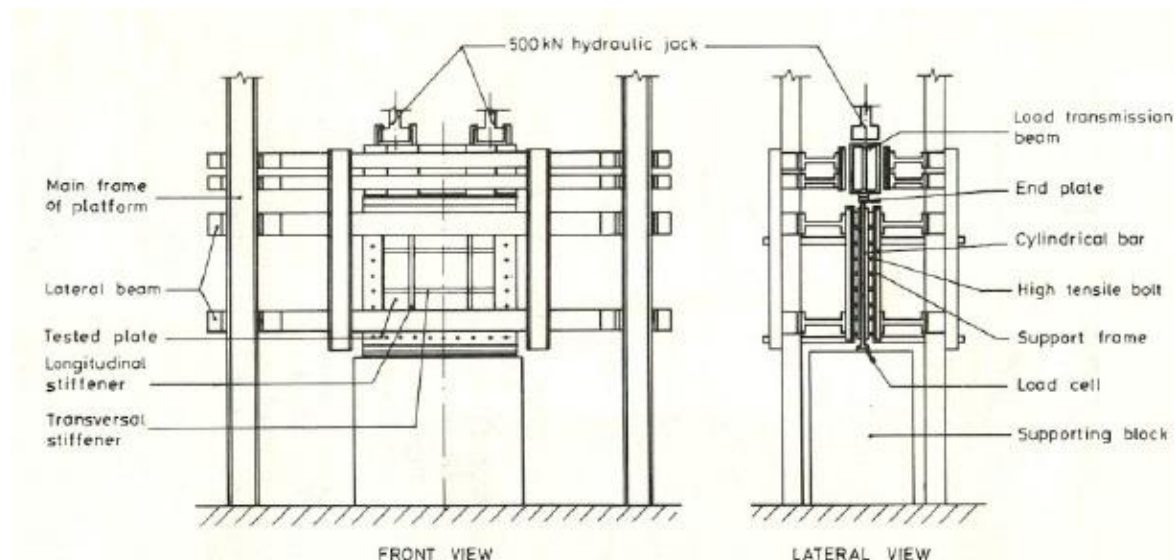


Figure 5: Ghavami's Testing Rig

supported but were very closely discretized simply supported and those with transversal stiffeners had continuously simply supported boundary conditions. A summary of material properties and test results is given in Table3.

Table 1: Definition of Ghavami Test Models

Series No.	Definition	Test models
I	Unstiffened plate	P1, P2
II	Plate with one longitudinal stiffener of R, L and T cross-section	P1R, P1L, P1T
III	Plate with two longitudinal stiffeners of R, L and T cross-section	P2R, P2L, P2T
IV	Plate as series II but with addition of one transversal stiffener at the mid-span	P1R1T, P1L1T, P1T1T
V	Plate as series III but with addition of one transversal stiffener at the mid-span	P2R1T, P2L1T, P2T1T
VI	Plate as series II but with addition of two transversal stiffeners at 1/3 of span	P2R2T, P2L2T, P2T2T

Table 2: Dimensions of Plate and Stiffeners in Ghavami Test Models

Test model	Plate			Longitudinal stiffener				Transverse stiffener			
	L	b	t	t _w	h _w	t _f	b _f	t _{wt}	h _{wt}	t _{ft}	b _{ft}
	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm
P1	650	650	4.4	----	----	----	----	----	----	----	----
P1R	650	325	4.4	7.0	30.0	----	----	----	----	----	----
P1L	650	325	4.4	6.4	30.0	3.9	16.4	----	----	----	----
P1T	650	325	4.4	6.4	30.0	4.8	26.4	----	----	----	----
P2R	650	217	4.4	7.0	30.0	----	----	----	----	----	----
P2L	650	216	4.4	6.4	30.0	19.5	16.4	----	----	----	----
P2T	650	217	4.4	6.4	30.0	20.0	26.4	----	----	----	----
P2	650	650	4.8	----	----	----	----	----	----	----	----
P1R1T	325	325	4.8	5.1	30.0	----	----	4.7	41.1	4.1	35.3
P1L1T	325	325	4.8	5.2	30.2	3.4	14.8	4.8	40.4	4.1	34.2
P1T1T	325	325	4.8	4.6	30.0	3.8	25.3	4.9	40.4	4.2	35.2
P2R1T	325	216	4.8	5.1	30.0	----	----	4.7	40.7	3.8	35.7
P2L1T	325	217	4.8	5.1	30.2	17.2	14.6	4.6	40.6	4.1	35.9
P2T1T	325	217	4.8	4.7	28.8	13.5	25.0	4.7	39.6	4.1	34.8
P2R2T	216	216	4.8	5.0	30.1	----	----	4.7	40.4	4.1	35.7
P2L2T	217	217	4.8	5.1	30.0	17.0	14.9	4.7	40.6	4.1	35.5
P2T2T	216	216	4.8	4.6	29.8	13.0	24.8	4.8	40.6	4.1	35.5

The testing rig was constructed within the Structural and Material Laboratory of PUC-Rio and is shown in Figs. 5 and 6. Out of plane deflections of plates and stiffeners were measured principally by mechanical dial gauges fixed at specific points mounted on the testing rig, as shown in Figure 6. In all models electrical linear strain gauges or rosettes measured the strains. More details on the test rig, test models and the process of the tests can be found in reference [14]. In each test the maximum ultimate collapse stress σ_{ult} was calculated by dividing the ultimate load P_u

to the overall cross-section of the plate A_p and stiffeners A_s as given by eqn (1):

$$\sigma_{ult} = P_u / (A_p + A_s) \dots\dots\dots (1)$$

The squash load P_{sq} was calculated by multiplying the yield stress of the plate σ_{Yp} and the stiffener σ_{Ys} with their appropriate cross-section areas as eqn (2):

$$P_{sq} = \sigma_{Yp} A_p + \sigma_{Ys} A_s \dots\dots\dots (2)$$

The test results together with those of maximum initial W_0 , final W_{max} deflections and inplane shortening U_{max} are given in Table 3.

Table 3: Summary of Material Properties and Results For Ghavami Test Models

Test model	Material properties			Measured deflection	Maximum deflection	Maximum shortening	Collapse stress
	E	σ_{Yp}	σ_{Ys}	W_o / t	W_{max} / t	U_{max} / t	$\sigma_{ult} / \sigma_{Yp}$
	$MPa \times 10^5$	MPa	MPa	%	%	%	%
P1	1.81	218	----	61	278	0.38	42.2
P1R	1.81	218	390	69	188	0.31	70.2
P1L	1.99	227	270	36	123	0.34	66.5
P1T	1.99	227	170	9	33	0.27	60.0
P2R	1.95	224	390	25	117	0.41	66.0
P2L	2.21	223	270	19	142	0.30	74.0
P2T	2.21	223	270	3	128	0.37	74.0
P2	1.78	220	----	20	121	0.40	48.2
P1R1T	1.85	219	326	21	123	0.33	74.0
P1L1T	1.91	225	326	27	27	0.48	71.1
P1T1T	1.75	219	273	33	121	0.33	72.1
P2R1T	1.75	219	326	70	52	0.64	88.6
P2L1T	1.89	227	326	40	33	0.60	84.6
P2T1T	1.78	220	273	23	30	0.51	89.1
P2R2T	1.91	225	326	32	18	0.63	86.2
P2L2T	1.89	227	326	28	53	0.56	97.4
P2T2T	2.09	218	273	32	16	0.51	103.2



Figure 6: Stiffened Plate Model Positioned In the Ghavami's Testing Rig

TANAKA & ENDOS' EXPERIMENTS

Tanaka & Endo [15] carried out a series of experimental and numerical investigations on the ultimate compressive strength of plates having three and two flat-bars stiffeners welded longitudinally and transversally respectively. A total of 12 tests were performed. The test specimen was designed so that the longitudinally stiffened plates located in the middle of whole test specimens could fail. The test specimens were intended to fail by local plate buckling or tripping of longitudinal stiffeners. A typical test rig from the Tanaka & Endo study is shown in Figure 7. A stiffened plate model positioned in their testing rig is presented schematically in Figure 8. To account for the effect of adjacent panels on the collapse behaviour of central panel, three-span models with

two adjacent (dummy) stiffened panels and supported by two transverse frames were employed. The thickness of plate and stiffeners in two adjacent panels was 1.2-1.3 times that of plate and stiffeners in the central panel. Table 4, where $a=1080\text{mm}$ is the span length of the plate with average plate thickness between $t=4.38\text{mm}$ to $t=6.15\text{mm}$, represents geometric and material properties for the Tanaka & En'os' test structures. The boundaries of stiffened plates were continuously simply supported and the in-plane axial compression load was applied longitudinally. The maximum measured initial deflections in the plate were ranging between 0.1-0.4 mm. The ultimate collapse strength and squash load were calculated in the same manner using

equations 1 and 2 as those considered by Ghavami [14].

Table 4: Geometric And Material Properties of Tanaka & Endo Tests

Structure No.	a (mm)	b (mm)	t (mm)	h_w (mm)	t_w (mm)	$\frac{h_w}{t_w}$	A_{03} (mm)	σ_{Yp} (MPa)	σ_{Ys} (MPa)	E (GPa)
D0	1080	1440	6.15	110.0	9.77	11.26	0.101	234.2	287.1	205.8
D0A		1440	5.65	110.0	10.15	10.84	0.250	249.9	196.0	205.8
D1		1200	5.95	110.0	10.19	10.79	0.143	253.8	250.9	205.8
D2		1560	5.95	110.0	10.19	10.79	0.288	253.8	250.9	205.8
D3		1440	5.95	103.5	11.84	8.74	0.312	253.8	326.3	205.8
D4		1440	5.95	118.5	7.98	14.85	0.119	253.8	284.2	205.8
D4A		1440	5.65	118.5	8.08	14.67	0.379	249.9	274.4	205.8
D10		1200	4.38	65.0	4.38	14.84	0.515	442.0	442.0	205.8
D11		1200	4.38	90.0	4.38	20.55	0.503	442.0	442.0	205.8
D12		1440	4.38	65.0	4.38	14.84	0.523	442.0	442.0	205.8

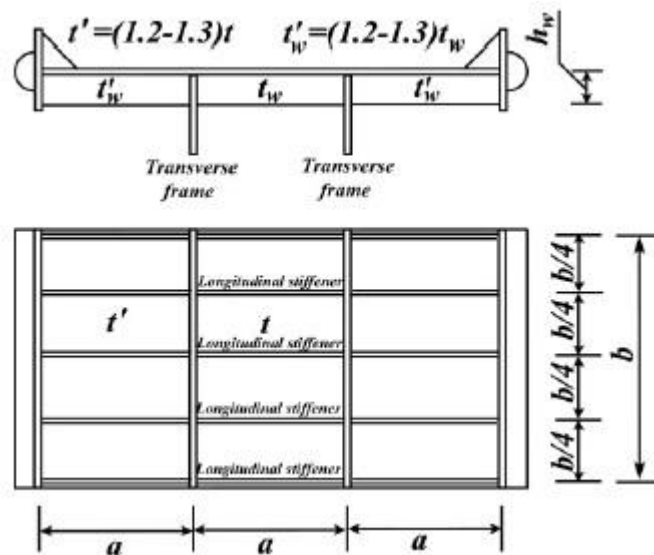


Figure 7: Tanaka & Endo's Test Model

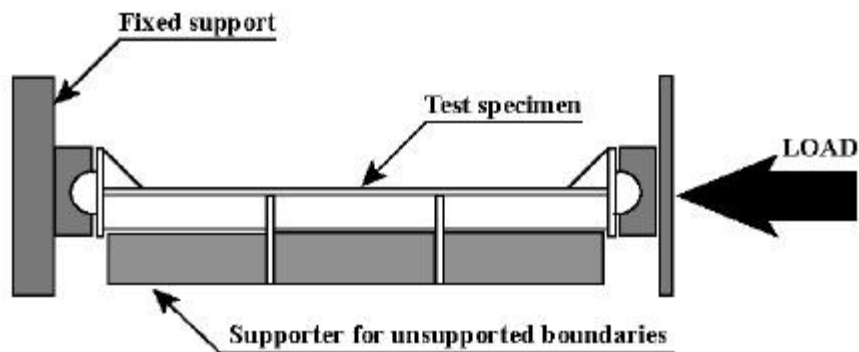


Figure 8: Stiffened Plate Model Positioned In The Tanaka & Endo's Testing Rig

FINITE ELEMENT SIMULATIONS

Since the test specimens in all above-reported experiments, had large deflections and plastic deformations, finite element analyses had to be performed using the software offering combined geometrical and material non-linear capabilities. In this study, the commercially available finite element code, ANSYS [16] was adopted. In the control menu of ANSYS solver, the options of “large deflection” and “arc-length method” are activated. The arc length method is used to trace the non-linear large deflection response of the models.

Shell Element Formulation

Both plate and stiffeners are modelled using SHELL43 elements selected from ANSYS library of elements. The SHELL43 element in Figure 9 is a so-called plastic large strain element and categorised in the family of four-node quadrilateral elements. Each node has three translational degrees of freedom in the nodal x, y and z directions as well as three rotational degrees of freedom about the nodal x, y and z-axes. The chosen element allows for elastic, perfectly plastic, with strain hardening or strain softening, large strain and large deflection response [16].

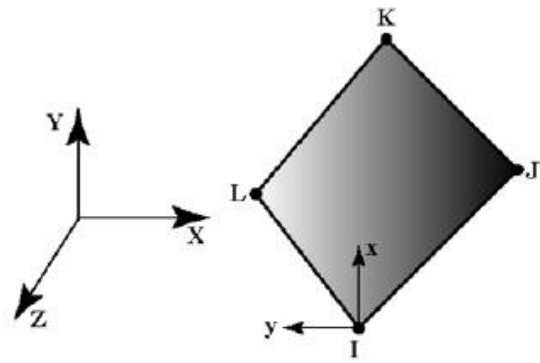


Figure 9: Shell43 Element of The ANSYS FEM Program

Finite Element Mesh And Boundary Conditions

A convergence study indicated that in the finite element mesh of isotropic and stiffened plates respectively, assuming 10×10 (a / b) mesh divisions along local plate panels and 10 mesh divisions across them is sufficient to capture accurately the buckling and plastic collapse behaviour. Respectively a and b represent the length and breadth of local plate panels. In order to model the stiffener's web and flange, respectively 6 to 7 and 5 to 6 elements are sufficient. However, the purpose of this study was to simulate the testing results and finer meshes were therefore used. In the case of Tanaka & Endo tests, to reduce the number of mesh divisions and speed up the time of analysis, a rational assumption was

made. The transverse stiffeners or frames for the case of Tanaka & Endo tests were not modelled for simplicity; instead the nodes on the line of attachment of the transverse stiffeners were constrained from translational movement out of plate plane. Furthermore, the translational movements of these nodes along the axis perpendicular to the line of attachment of

transverse stiffeners were coupled with each other. Transverse frames were modelled in the case of Ghavami's tests. In both Ghavami and Tanaka & Endo's tests, the stiffened plates were loaded in axial compression along the stiffeners. Also in their tests the simply supported boundary conditions were assumed in the models. Figures 10 and 11 show typical

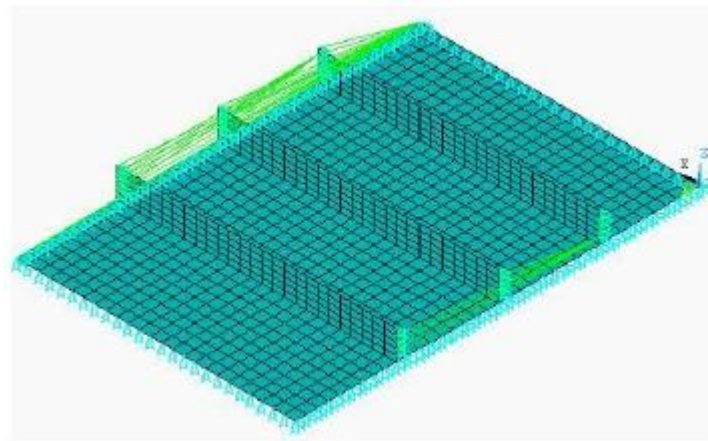


Figure 10: Finite Element Model of Tanaka & Endo's Test Specimens

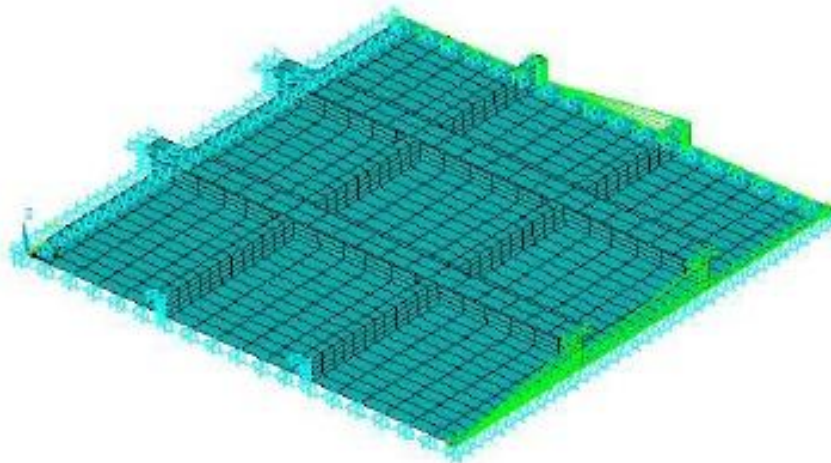


Figure 11: Finite Element Model Of Ghavami's P2L2T Test Specimen

LARGE DEFLECTION BEHAVIOUR OF STIFFENED PLATES SUBJECTED TO COMBINED IN-PLANE COMPRESSION AND LATERAL PRESSURE

For the stiffened plates in the bottom structure of ships, the basic load case for buckling design consists of the following loads applied simultaneously (Figure 19):

- longitudinal compression arising from the overall hull girder bending,
- transverse compression arising from the bending of double bottom under lateral pressure, and
- Local bending arising from the direct action of lateral pressure.

The continuous plate was assumed to be simply-supported along the stiffener lines with no out-of-plane deflection. In reality, however, the stiffener is also subjected to lateral pressure, and it may collapse prior to the failure of the panels. The focus of the present chapter is concentrated on the buckling and plastic collapse behaviour of continuously stiffened plates subjected to combined biaxial compression and lateral pressure with the main objective of identification of the collapse modes of the plates subjected to mentioned combination of loading condition.

A series of elasto-plastic large deflection FEM analyses is performed on continuous stiffened plates with flat-bar, tee-bar, and angle-bar stiffeners of the same flexural rigidity. The buckling/plastic collapse behaviour and ultimate strength of stiffened plates are hereby assessed so that both the material and geometrical nonlinearities are taken into account.

Local plate panels with length, a , of 2400 mm and breadth, b , of 800 mm are considered, and their thickness, t , changes from 13mm, 15mm, and 20 mm. Yield stress of the material, σ_Y , is taken as 313.6 MPa, and bilinear stress-strain relationship is assumed with the kinematical strain-hardening rate of $E/65$, where E is Young's modulus of the material. E is considered as 205.8GPa. The cross-sectional geometries of stiffeners are given in Table 8. In each group, the stiffeners have the same moment of inertia. A triple span-double bay model is applied for the analysis of buckling/plastic collapse behaviour of continuous stiffened plate with symmetrical stiffeners (ABDC in Figure 20). When a stiffener has an unsymmetrical geometry, a triple span-triple bay model is used (ABFE in Figure 12) [23].

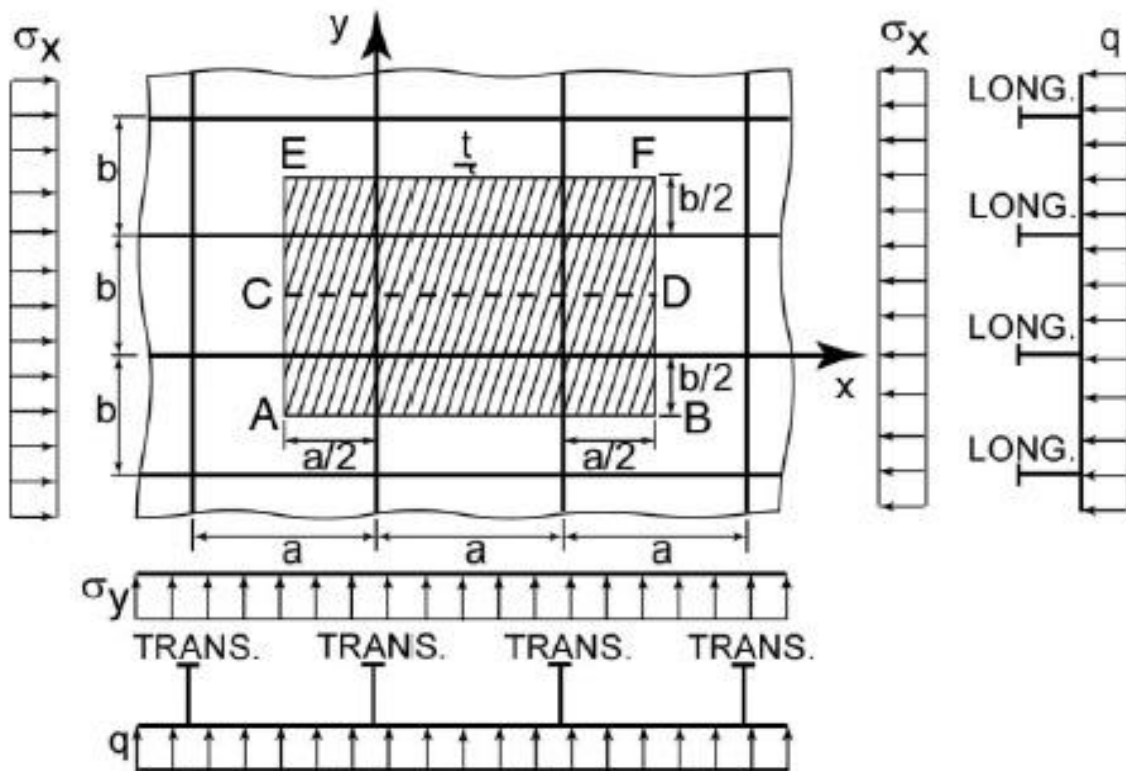


Figure 12: Basic Loads Applied On Ship Stiffened Plates

The considered boundary conditions are as follows:

- Periodically continuous conditions are imposed at the same y -coordinate along the transverse edges (i.e. along AC and BD in double bay model and along AE and BF in triple bay model).
- Symmetry conditions are imposed along the longitudinal edges of double bay model (i.e. along AB and CD). But
- periodically continuous conditions are defined at the same x -coordinate along the longitudinal edges of triple bay model (i.e. along AB and EF).

- modelled, the out-of-plane deformations of plate and stiffener are restrained along the junction lines of them and the transverse frame.
- To consider the plate continuity, in-plane movement of the plate edges in their perpendicular directions is assumed to be uniform.

The lateral pressure ranging from 0 to 60 metres water head initially is applied up to a specified value always perpendicularly to the plate surface. Then biaxial compression is exerted proportionally by uniform forced displacements.

CONCLUSION

Basing the results of this chapter on the analysis of 29 experimental investigation, on stiffened steel plates subjected to uniform axial compression load up to final failure, by the Finite Element program ANSYS, the following conclusions may be drawn. The selected element SHELL43, could trace full-range, elastic-plastic behaviour of the stiffened plates. The capability of the non-linear FEM to perform the analysis of stiffened plates has been demonstrated through the accurate simulation of the Ghavami and Tanaka & Endo tests.

Although some simplifying assumptions for the simulation of initial imperfections and residual welding stresses were made for reducing the calculation volume and speeding up the analysis, the accuracy of the collapse load obtained through FEM simulations is relatively in good consistency with the test results. The differences were higher in cases of not having perfect simply supported boundary conditions as in series II and III of Ghavami's test. It was shown also, that obtaining deflection mode is possible at any step of loading. This allows predicting the local buckling of stiffened plates with relatively good precision. For small value of lateral pressure, the local panel and

stiffener tend to collapse in a simply supported mode. With an increase in the applied pressure, they are likely to fail in a clamped mode. Angle-bar stiffener has larger stiffening effects than those of flat-bar and tee-bar stiffeners having the same flexural rigidity, from the view point of ultimate strength.

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