Conducting Materials in Electrostatic Fields - Charge Distribution and Equilibrium

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Abstract

In this paper, the behavior of conducting materials in electrostatic fields is studied. It is important to observe the equilibrium state of the electric charges when conducting materials are put through experiments to determine their usability in industrial applications. The relationship between charge distribution and its equilibrium state are studied with regards to time constant, also known as the relaxation time. As the electric field in a conducting material is zero when the charges flowing inside it reach their equilibrium, some quick observations are made to understand the behavior of conducting materials under different charges. Time factor is an important consideration in this. .

Keywords: *Electrostatic Fields, Charge Distribution and Equilibrium, Laplace's and Poisson's Equations*

INTRODUCTION

A conducting material is one which allows free movement of electric charge within it on a time scale which is short compared with that of the problem. Under this definition metals are always conductors but some other materials which are insulators on a short time scale may allow a redistribution of charge on a longer one. They may be regarded as conducting materials in electrostatic problems if we are prepared to wait for long enough for the charges to reach equilibrium. The charge distribution tends to equilibrium as exp(– t/τ), where the time constant τ is known as the relaxation time. Once the charges have reached equilibrium there can be no force acting on them and the electric field within the material must be zero.

INDUCED CHARGE

Some typical values of charge distribution are:

Copper l.5 × l0-19 s

Distilled water l0-6 s

Fused quartz l06 s

When an uncharged conducting body is placed in an electric field, the free charges within it must redistribute themselves to produce zero net field within the body. Consider, for example, a copper sphere placed in a uniform electric field. The copper has within it about 1029 conduction electrons per cubic meter, and their charge is balanced by the equal and opposite charge of the ionic cores fixed in the crystal lattice. The available conduction charge is of the order of 1010 C m-3, and only a tiny fraction of this charge has to be redistributed to cancel any practicable electric field. This redistribution gives rise to a surface charge, somewhat as shown in Fig. 1, whose field within the sphere is exactly equal and opposite to the field into which the sphere has been placed.

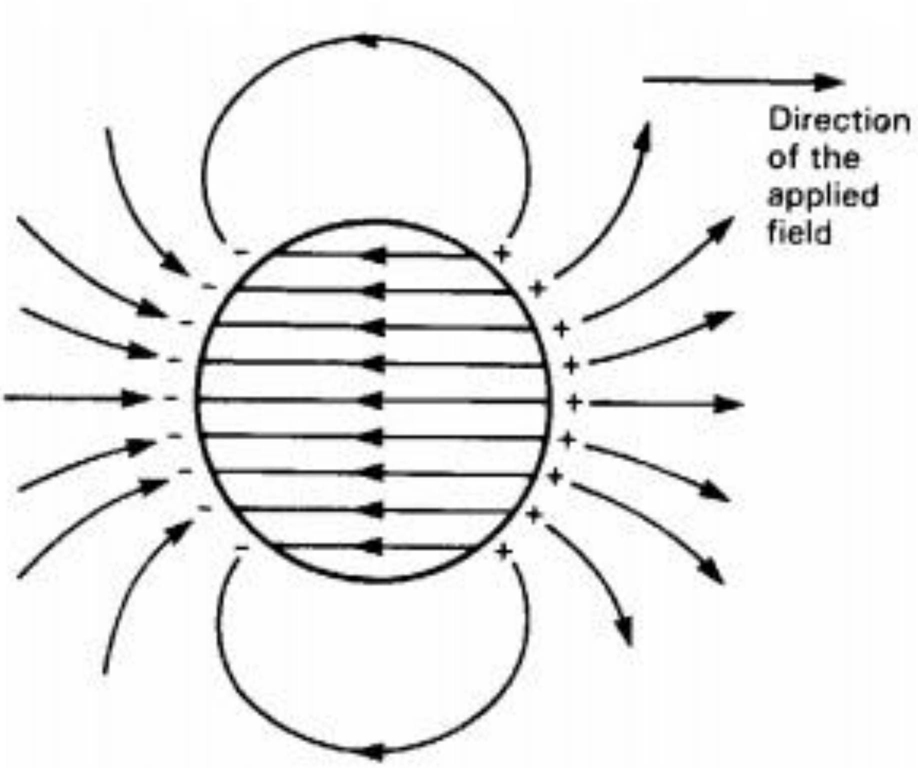


Fig. 1 The field pattern of the charge induced on a conducting sphere placed in a uniform electric field.

This surface charge is known as induced charge. It is important to remember that the positive and negative charges balance so that the sphere still carries no net charge. The complete solution to the problem is obtained by superimposing the original uniform field on that shown in Fig.1 to give the field shown in Fig. 2. Note that the flux lines must meet the surface of the sphere at right angles because the surface is an equipotential.

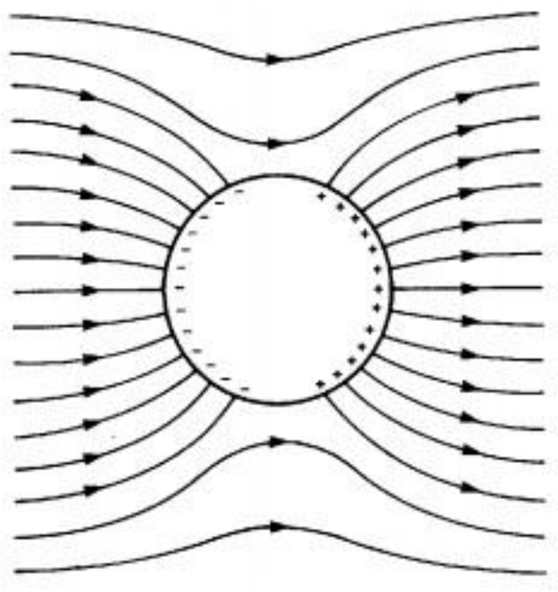


Fig. 2 The field pattern around a conducting sphere placed in a uniform electric field. This pattern is obtained by superimposing the field of the induced charges (shown in Fig. 2) on the uniform field.

Not only is there no electric field within a conducting body, but there is also no field within a closed conducting shell placed in an electric field. To prove this, consider the figure on the right, which shows a closed conducting shell S_1 there must be other equipotentials such as S_2 lying wholly within S_1. Now the interior of the shell contains no free charge, so, applying Gauss' theorem to S_2, the flux of E out of S_2 is zero. But, since it has been postulated that S_2 is an equipotential surface, this can be true only if E is zero everywhere on it and the potential of S_2 is the same as that of S_1.

A closed hollow earthed conductor can therefore be used to screen sensitive electronic equipment from electrostatic interference. The screening is perfect as long as there are no holes in the enclosure, for example to allow wires to pass through. Even when there are holes in the enclosure the screening can still be quite effective. When the electric field varies with time other screening mechanisms come into play and the screening is no longer so perfect.

THE METHOD OF IMAGES

We have already seen that the electric field produced by a known distribution of charges can be calculated in simple cases, by the application of Gauss' theorem and the principle of superposition. In most practical problems, however, the charge is unknown and the problem is specified in terms of the potentials on electrodes. Simple problems of this type can be solved by the use of Gauss' theorem if it is possible to make assumptions about the distribution of charges from the symmetry of the problem.

If an uncharged, isolated, conducting sheet is placed in an electric field, then equal positive and negative charges are induced on it. Normally this process requires currents to flow in the plane of the sheet, and the field pattern is changed so that the sheet becomes an equipotential surface. If, however, the sheet is arranged so that it coincides with an equipotential surface, the direction of current flow is normal to the plane of the sheet and the two surfaces become oppositely charged. If the sheet is thin, the separation of the positive and negative charges is small and the field pattern is not affected by the presence of the sheet. This fact can be used to extend the range of problems which can be solved by elementary methods. For example, a conducting sheet can be placed along the equipotential. It screens the two charged wires from each other so that either could be removed without affecting the field pattern on the other side of the sheet. Thus the field pattern between a charged wire and a conducting plane is just half of that of a pair of oppositely charged conducting wires.

The field between a charged wire and a conducting plane can be found by reversing the train of thought. We note that an image charge can be placed on the opposite side of the plane to produce a field which is the mirror image of the original field. The image charge is equal in magnitude to the original charge, but has the opposite sign. The plane is an equipotential surface in the field of the two charges, so it can be removed without altering the field pattern. The problem is then reduced to the superposition of the fields of the original and image charges. This method is known as the method of images. It can be applied to the solution of any problem involving charges and conducting surfaces if a set of image charges can be found such that the equipotentials in free space of the whole set of charges coincide with the conducting boundaries.

LAPLACE'S AND POISSON'S EQUATIONS

The method described in the previous section has been applied with ingenuity to a wide variety of problems whose solutions can be looked up when required.

Unfortunately engineers are not free to choose the problems they wish to solve, and the great majority of practical problems cannot be solved by elementary methods. Figure 3 shows a typical problem: an electron gun of the kind used to generate the electron beam in a microwave tube for satellite communications.

In this case the field problem and the equations of motion of the electrons must be solved simultaneously because the space charge of the electrons affects the field solution. A general method which can be used, in principle, to solve any problem is obtained by combining Equations to give

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial zy^2}=-\frac{\rho}{\varepsilon_0} \quad (1)$$

This is known as Poisson's equation. It can also be written as –

$$\nabla^2 V=-\frac{\rho}{\varepsilon_0} \quad (2)$$

where ∇2 is given, in rectangular Cartesian coordinates, by

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} \quad (3)$$

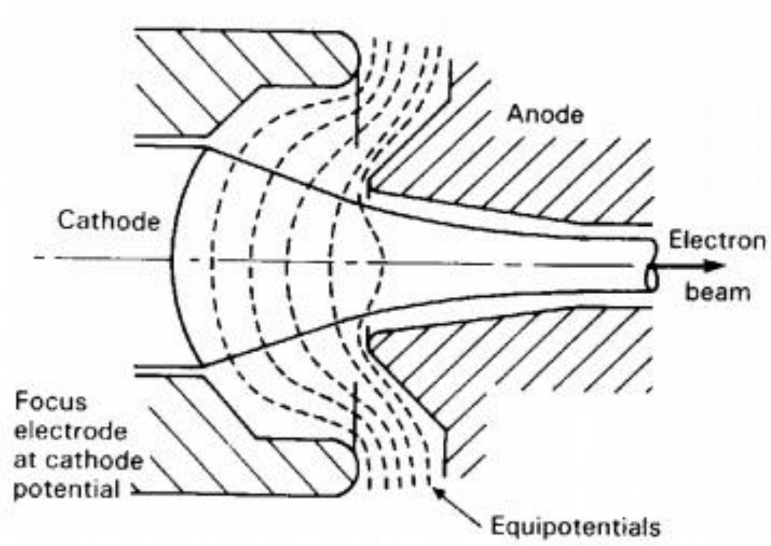


Fig. 3 The arrangement of a typical high-power electron gun, such a gun might produce a 50 mA electron beam 2 mm in diameter for a potential difference between cathode and anode of 5 kV.

This equation has been solved for a very wide range of boundary conditions by analytical methods employing a variety of coordinate systems and by the special method known as conformal mapping, which applies to two-dimensional problems. These solutions can be looked up when they are required. Cases whose solutions are not available in the literature must, in nearly every case, be solved by numerical methods. When free charges are present in a problem it is necessary to use Poisson's equation as the basis of either an analytical or a numerical solution. There are only a few cases which can be solved analytically.

In every kind of active electronic device electric fields are used to control the motion of charged particles. The methods described here can be applied to the motion of charged particles in vacuum. When the charge densities are small it is possible to calculate the electrostatic fields, neglecting the contributions of the charges to them, and then to integrate the equations of motion of the particles. At higher charge densities the fields are affected by the space charge and it is necessary to find mutually consistent solutions of Poisson's equation and the equations of motion. The motion of charge carriers in semiconductor devices such as transistors requires knowledge of the fields in material media.

THE FINITE DIFFERENCE METHOD

The simplest numerical method for solving field problems is the finite difference method. In this method a regular rectangular mesh is superimposed upon the problem. The real continuous variation of potential with position is then approximated by the values of the potential at the intersections of the mesh lines. Figure 4 shows a small section of a two-dimensional mesh with a spacing h in each direction and the electrostatic potentials at the mesh points.

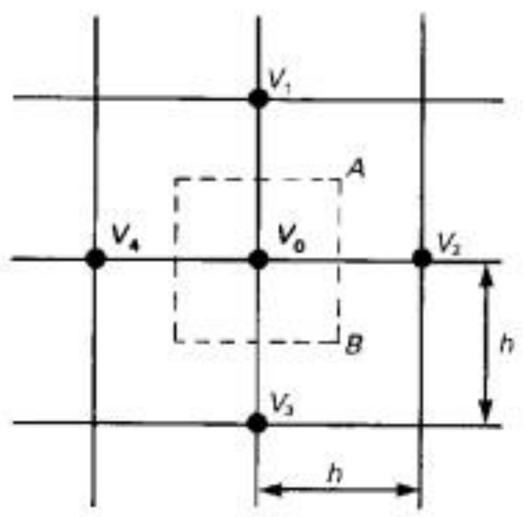


Fig. 4 Basis of the finite difference calculation of potential

To find an approximate relationship between the potentials shown we apply Gauss' theorem to the surface shown by the broken line. The component of the electric field normal to the section AB of the surface is given approx approximately by

$$E_{nAB}=(V_0-V_2)/h \quad (4)$$

The flux of E through unit depth of the face AB is therefore

$$\Phi_{AB} = E_{nAB}h = V_0 - V_2 \quad (5)$$

The component of the electric field normal to the section AB of the surface is given approx.

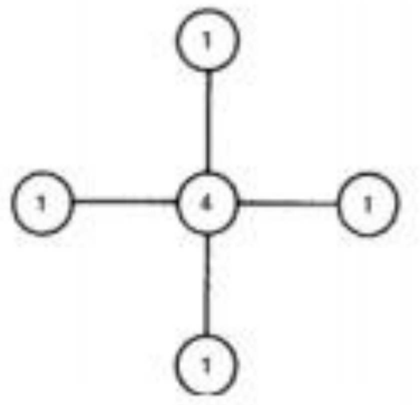
If the Gaussian surface does not enclose any charge the net flux of E out of it must be zero so that

$$4V_0 - V_1 - V_2 - V_3 - V_4 = 0$$

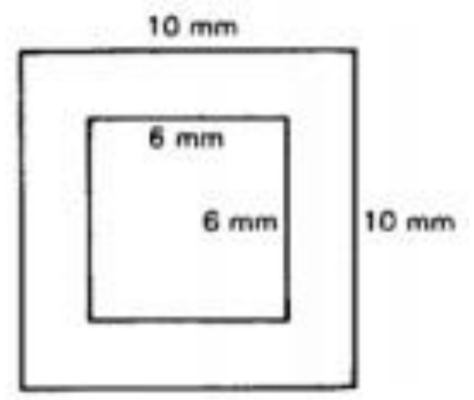
Or

$$V_0 = ¼ (V_1 + V_2 + V_2 + V_3) \quad (6)$$

Thus, if we know the potentials at points 1 to 4 approximately, we can use Equation (6) to obtain an estimate of V_0. Because the errors in the four potentials cancel each other out to some extent, and because the resulting error is divided by 4, the error in the value of V0 is normally less than the errors in the potentials used to calculate it. Equation (6) is conveniently summarized by the diagram on the right.



This method can be used to find the fields around two-dimensional arrangements of electrodes on which the potentials are specified such as the concentric square tubes shown in the figure on the right. The method can be implemented on a spreadsheet as follows:



a) A uniform square mesh is defined such that the electrodes coincide with mesh lines. The mesh spacing is chosen so that it is small enough to provide a reasonably detailed approximation to the fields whilst not being so small that the computational time is very large.

b) Cells of the spreadsheet are marked out such that one cell corresponds to each mesh point. The symmetry of the problem can be used to reduce the number of cells required. Thus, for the geometry shown above it is sufficient to find the solution for one quadrant of the problem.

c) The electrode potentials are entered into the cells corresponding to the electrodes and the formula in Equation (1.30) is entered into all the other cells. When symmetry has been used to reduce the size of the problem the formulae in the cells along symmetry boundaries make use of the fact that the potentials on either side of the boundary are equal.

d) The formulae in the cells are then applied repeatedly (a process known as iteration) until the numbers in the cells cease to change. To do this the calculation options of the spreadsheet must be set to permit iteration. The final numbers in the cells are then approximations to the potentials at the corresponding points in space.

e) From this solution the equipotential curves can be plotted by interpolation between the potentials at the mesh points and the field components can be calculated at any mesh point.

The method can be applied to more complicated problems including those with curved electrodes which do not fit the mesh and three-dimensional problems. Further information can be found in the literature.

CONCLUSION

In this paper, starting from the inverse square law of force between two charges, we derived a range of methods for solving practical problems involving electric fields in free space. The concepts of electric field, flux density and potential have been shown to be useful for these purposes. The ideas contained in in this paper find their direct application in problems about voltage breakdown between electrodes in air and those dealing with the motion of charged particles in vacuum.

The very limited range of problems which can be solved by elementary methods can be extended by the use of the principle of superposition and the method of images.

In most real problems, however, the electric field can be calculated only by solving Laplace's or Poisson's equations. Cases which have not been solved before generally have to be tackled using numerical methods.

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