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Bending Analysis of Thick Isotropic Beam by Using 5th Order Shear Deformation Theory

***S. P. Patil*[1]*, U. B. Kalwane*[2]*, J. P. Bhandari*[3]**
Department of Civil Engineering
Shreeyash College of Engineering and Technology, Aurangabad (M. S.), India
Corresponding Author's email: sppatilcivilengg@gmail.com

Abstract

A 5th order shear deformation theory considering transverse shear deformation effect is presented for static flexure analysis of fixed isotropic beam. The assumed displacement field accounts for non-linear variation of in-plane displacements as well as transverse displacement through the beam thickness. The condition of zero transverse shear stresses on the top and bottom surface of beam is satisfied. Hence the present formulation does not require the shear correction factor generally associated with the first order shear deformable theory. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. Closed form analytical solutions for fixed isotropic thick beams subjected to various loads are obtained. Numerical results for static flexure analysis include the effects of side to thickness ratio and beam aspect ratio for fixed isotropic beams. The results of present theory are in close agreement with those of higher order shear deformation theories.

Keywords: *Thick isotropic beam, 5th order shear deformation theory, flexure, displacement, shear stress*

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INTRODUCTION

Beams are the important structural member that are mostly used in various engineering disciplines such as aerospace, civil, marine and mechanical engineering and many more. The transverse shear deformation effects are more pronounced in shear flexible beams which may be made up of isotropic, orthotropic, anisotropic or laminated composite materials. To address the correct structural behavior of structural elements made up of these materials; development of refined theories, which consider refined effects in static analysis of structural elements, becomes necessary. The wide spread use of shear flexible materials has stimulated interest in the accurate prediction of structural behavior of thick beams. Thick beams, either isotropic or anisotropic, basically form two and three dimensional problems of elasticity theory. Reduction of these problems to the corresponding one and two dimensional approximate problems for their analysis has always been the main objective of research workers. The shear deformation effects are more pronounced in case of thick beams when subjected to lateral loads than in the thin beams under similar loading. These effects are neglected in elementary theory of beam. To describe the correct bending behavior of thick beams including shear deformation effects and the associated cross sectional warping, shear deformation theories are required.

A comprehensive review of refined shear deformation theories for isotropic beams is given by Ghugal and Shimpi [1] and further recently reviewed by Sayyad and Ghugal [2]. It is well-known that the classical beam theory based on Navier-Kirchhoff hypothesis under predicts deflections and over predicts natural frequencies and buckling loads due to the neglect of transverse shear deformation effects. This theory is known as classical beam theory. Timoshenko and Woinowsky-Krieger [3] presented the static flexure and buckling solutions various thin plates subjected various loading and boundary conditions.

The progress in the theory of beams and plates formulation made in 1789-1988 has been carefully reviewed by Jemielita [4]. Refined theories, mainly due to Levy [5], Reissner [6], Hencky [7], Mindlin [8] and Kromm [9] are improvements over the classical beam theory in which the effect of transverse shear deformation is included. Reissner's theory is stress based and Hencky, Mindlin theories are displacement

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based in which the displacements are expanded in powers of the thickness of beam.

These theories are well-known as first order shear deformation theories (FSDTs) in the literature and widely referred to as Reissner-Mindlin beam theory. These theories, however, do not satisfy the shear stress free boundary conditions on the surfaces of the beam and require shear correction factors to consider strain energy due shear deformation appropriately. The deficiencies in classical and first order shear deformation theories led to the development of higher order or equivalent shear deformation theories.

The first refined (higher order) theories are due to Levy's paper of 1877 [5]. Hundred years later in 1977 Lo, Christensen and Wu [10, 11] developed a consistent higher order theory, based on Levy's kinematic hypothesis, for homogeneous and laminated plates including effects of transverse shear deformation, transverse normal strain and a nonlinear distribution of the in-plane and transverse displacements with respect to the thickness coordinate. Lo et al. theory contains eleven unknown displacement variables. It is basically a third order unconstrained shear deformation theory which includes transverse shear and normal deformation effects. Many higher order shear deformation theories are developed later based on Lo et al., theory which are either constrained or unconstrained theories.

These theories are reviewed in great details in Ref. [1] and [2]. Kant [12] studied the bending behavior of a thick homogeneous and isotropic rectangular plate using higher order shear and normal deformation theory in conjunction with segmentation method. A linear elastic analysis is presented. Kant and Swaminathan [13] presented a review of various unconstrained models derived from Lo et al. third order theory for the static analysis of composite beams. The third-order parabolic shear deformation theories satisfying shear stress free boundary conditions on the bounding planes of beam are studied by many researchers and critically reviewed by Jemielita [14].

Ghugal and Dahake [15, 16] studied the flexure of thick beams using refined shear deformation theory. Dahake and Ghugal [17,18] presented trigonometric shear deformation theory for thick beams.

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This has motivated the authors of present paper to investigate the bending response of thick isotropic beams. In this paper a fifth order shear deformation theory with transverse normal strain effect is developed for the first time and accurate description transverse shear stresses are presented. The displacements and stresses obtained are compared in case of isotropic beam. The generalized theory formulation suitable for homogeneous, linearly elastic, isotropic thick beam is presented.

DEVELOPMENT OF THEORY

The beam under consideration as shown in Fig.1 occupies in Cartesian coordinate system the region:

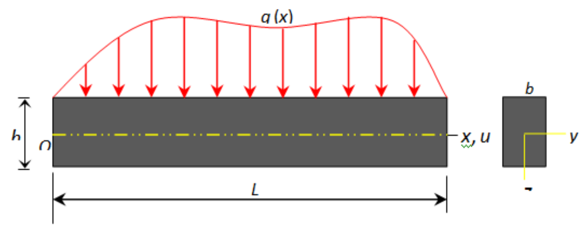

Fig. 1: Beam under bending in x-z plane

$$0 \le x \le L \;;\quad 0 \le y \le b \;;\quad -\frac{h}{2} \le z \le \frac{h}{2}$$

where x, y, z are Cartesian coordinates, L and b are the length and width of beam in the x and y directions respectively, and h is the thickness of the beam in the z-direction. The beam is made up of homogeneous, linearly elastic isotropic material.

2.1 The displacement field

The displacement field of the present beam theory is as given below:

$$u(x,z) = u_0 - z\frac{dw}{dx} + z\left[2 - \frac{4}{3}\left(\frac{z}{h}\right)^2 - \frac{16}{5}\left(\frac{z^2}{h^2}\right)^2\right]\phi(x) \qquad (1)$$

$$w(x,\, z) = w_b + w_s$$

Normal Strain:

$$\varepsilon_x = \frac{du}{dx} = -z\frac{d^2 w}{dx^2} + z\left(2 - \frac{4}{3}\left(\frac{z}{h}\right)^2 - \frac{16}{5}\frac{z^4}{h^4}\right)\frac{d\phi}{dx} \qquad (2)$$

Shear Strain:

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$$\gamma_{zx} = \frac{du}{dz} + \frac{dw}{dx} = \left(2 - 4\frac{z^2}{h^2} - 16\frac{z^4}{h^4}\right)\phi(x) \tag{3}$$

Stress-Strain Relationships

$$\begin{aligned} \sigma_x &= E\varepsilon_x \\ \tau_{zx} &= G\gamma_{zx} \end{aligned} \tag{4}$$

2.2 Governing Equations and Boundary Conditions

Using the expressions for strains and stresses (2) through (4) and using the principle of virtual work, variationally consistent governing differential equations and boundary conditions for the beam under consideration can be obtained. The principle of virtual work when applied to the beam leads to:

$$b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=+h/2}\left(\sigma_x \delta\varepsilon_x + \tau_{zx}\delta\gamma_{zx}\right)dx\,dz - \int_{x=0}^{x=L} q(x)\,\delta w\,dx = 0 \tag{5}$$

where the symbol denotes the variational operator. Employing Green's theorem in Eqn. (4) successively, we obtain the coupled Euler-Lagrange equations which are the governing differential equations and associated boundary conditions of the beam. The governing differential equations obtained are as follows:

$$EI\frac{d^4w}{dx^4} - \frac{12}{7}EI\frac{d^3\phi}{dx^3} - q(x) = 0 \tag{6}$$

$$\frac{12}{7}EI\frac{d^3w}{dx^3} - 2.96\,EI\frac{d^2\phi}{dx^2} + 2.4635\,GA\,\phi = 0 \tag{7}$$

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The associated variationally consistent boundary conditions obtained at the ends x = 0 and x = L is of following form:

$$\text{Either } V_x = EI\frac{d^3w}{dx^3} - \frac{12}{7}EI\frac{d^2\phi}{dx^2} = 0 \text{ or } w \text{ is prescribed} \tag{8}$$

$$\text{Either } M_x = EI\frac{d^2w}{dx^2} - \frac{12}{7}EI\frac{d\phi}{dx} = 0 \text{ or } \frac{dw}{dx} \text{ is prescribed} \tag{9}$$

$$\text{Either } M_s = \frac{12}{7}EI\frac{d^2w}{dx^2} + 2.96EI\frac{d\phi}{dx} = 0 \text{ or } \phi \text{ is prescribed} \tag{10}$$

where V_x, M_x are the shear force and bending moment resultants respectively analogous to elementary theory of beam bending and M_s is the moment resultant due to the effect of transverse shear deformation.

2.3 The general solution of governing equilibrium equations of the beam

The general solution for transverse displacement $w(x)$ and warping function $\phi(x)$ is obtained using Eqns. (6) and (7) using method of solution of linear differential equations with constant coefficients. Integrating and rearranging the first governing Eqn. (6), we obtain the following equation

$$\frac{d^3w}{dx^3} = \frac{12}{7}\frac{d^2\phi}{dx^2} + \frac{Q(x)}{EI} \tag{11}$$

where $Q(x)$ is the generalized shear force for beam and it is given by $Q(x) = \int_0^x q\,dx + C_1$.

Now second governing Eqn. (7) is rearranged in the following form:

$$\frac{12}{7}\frac{d^3w}{dx^3} - 2.96\frac{d^2\phi}{dx^2} = -\beta\phi \tag{12}$$

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A single equation in terms of is now obtained using Eqns (11) and (12) as:

$$\frac{d^2\phi}{dx^2} - \lambda^2 \phi = \frac{Q(x)}{\alpha EI} \tag{13}$$

Let, $A_0 = \frac{12}{7}, \quad B_0 = 2.96, \quad C_0 = 2.4635$

where constants α , β and λ in Eqns. (10) and (11) are as follows

$$\alpha = \left(\frac{B_0}{A_0} - A_0 \right), \quad \beta = \frac{C_0 GA}{A_0 EI}, \text{ and } \lambda^2 = \frac{\beta}{\alpha}$$

The general solution of Eqn. (11) is as follows:

$$\phi(x) = C_2 \cosh \lambda x + C_3 \sinh \lambda x - \frac{Q(x)}{\beta EI} \tag{14}$$

The equation of transverse displacement $w(x)$ is obtained by substituting the expression of ϕ (x) in Eqn. (12) and then integrating it thrice with respect to x. The general solution for $w(x)$ is obtained as follows:

$$EIw(x) = \iiint q\,dx\,dx\,dx + \frac{c_1 x^3}{6} + \left(\frac{A_0}{B_0}\lambda^2 - \beta \right)\frac{EI}{\lambda^3}\left(c_2 \sinh \lambda x - c_3 \cosh \lambda x \right) + c_4 \frac{x^2}{2} + c_5 x + c_6 \tag{15}$$

where, $C_1, C_2, C_3, C_4, C_5 \text{ and } C_6$ are arbitrary constants and can be obtained by imposing natural (forced) and / or geometric or kinematical boundary / end conditions of beam.

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Illustrative Example

In order to prove the efficacy of the present theory, the following numerical examples are considered. The following material properties for beam are used

$E = 210$ GPa, $\mu = 0.3$ and $\rho = 7800$ kg/m^3

where E is the Young's modulus, ρ is the density, and μ is the Poisson's ratio of beam material.

The kinematic and static (forced) boundary conditions associated with fixed beam bending problem is as follows:

$$\frac{dw}{dx} = \phi = w = 0 \text{ at } x = 0, L$$

Example Fixed Beam with Parabolic Load, $q(x) = q_0 \dfrac{x^2}{L^2}$

A fixed-fixed beam has its origin at left hand side support and is fixed at $x = 0$ and L. The beam is subjected to parabolic load, on surface z = $+h/2$ acting in the downward z direction with maximum intensity of load q_0.

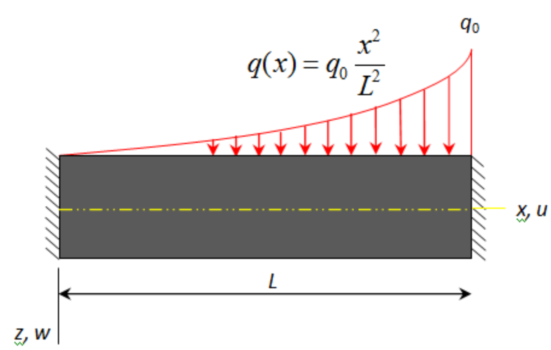

Fig. 2: Fixed beam with parabolic load

General expressions obtained for $w(x)$, $\phi(x)$, σ_x, τ_{zx}^{CR} and τ_{zx}^{EE} are as follows:

$$w(x) = \frac{q_0 L^4}{120 EI}\left\{\begin{array}{l} \dfrac{1}{3}\dfrac{x^6}{L^6} - \dfrac{4}{3}\dfrac{x^3}{L^3} + \dfrac{x^2}{L^2} - \dfrac{2}{3}\dfrac{A_0^2}{C_0}\dfrac{E}{G}\dfrac{h^2}{L^2}\left[\dfrac{1}{2}\dfrac{x^2}{L^2} - \dfrac{x}{L} + \dfrac{1 + \sinh\lambda x - \cosh\lambda x}{\lambda L}\right] \\ -\dfrac{B_0}{C_0}\dfrac{E}{G}\dfrac{h^2}{L^2}\left(\dfrac{5}{6}\dfrac{x^4}{L^4} - \dfrac{5}{3}\dfrac{x^2}{L^2}\right) \end{array}\right\} \tag{16}$$

$$\phi(x) = -\frac{1}{15}\frac{q_0 L}{GA}\frac{A_0}{C_0}\left(5\frac{x^3}{L^3} - 1 - \sinh\lambda x + \cosh\lambda x\right) \tag{17}$$

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$$u = \frac{q_0 h}{Eb}\left\{\begin{array}{l} -\dfrac{z}{h}\dfrac{1}{10}\dfrac{L^3}{h^3}\left[\begin{array}{l} 2\dfrac{x^5}{L^5} - 4\dfrac{x^2}{L^2} + 2\dfrac{x}{L} - \dfrac{2}{3}\dfrac{A_0^2}{C_0}\dfrac{E}{G}\dfrac{h^2}{L^2}\left(\dfrac{x}{L} - 1 + \cosh\lambda x - \sinh\lambda x\right) \\ -\dfrac{10}{3}\dfrac{B_0}{C_0}\dfrac{E}{G}\dfrac{h^2}{L^2}\left(\dfrac{x^3}{L^3} - \dfrac{x}{L}\right) \end{array}\right] \\ -\dfrac{z}{h}\dfrac{A_0}{C_0}\dfrac{E}{G}\dfrac{L}{h}\dfrac{1}{15}\left(5\dfrac{x^3}{L^3} - 1 + \cosh\lambda x - \sinh\lambda x\right)\left(1 - \dfrac{4}{3}\dfrac{z^2}{h^2}\right) \end{array}\right\} \tag{18}$$

$$\begin{aligned}\sigma_x = \frac{q_0}{b}&\left\{-\frac{z}{h}\frac{L^2}{h^2}\left[\begin{array}{l} \left(\dfrac{x^4}{L^4} - \dfrac{4}{5}\dfrac{x}{L} + \dfrac{1}{5}\right) - \dfrac{E}{G}\dfrac{B_0}{C_0}\dfrac{h^2}{L^2}\left(\dfrac{x^2}{L^2} - \dfrac{1}{3}\right) \\ -\dfrac{2}{30}\dfrac{A_0^2}{C_0}\dfrac{E}{G}\dfrac{h^2}{L^2}\left(\lambda L\left(\sinh\lambda x - \cosh\lambda x\right) + 1\right) \end{array}\right]\right\} \\ &+ \frac{1}{15}\frac{z}{h}\frac{E}{G}\frac{A_0}{C_0}\left(1 - \frac{4}{3}\frac{z^2}{h^2}\right)\left(\lambda L\sinh\lambda L - \lambda L\cosh\lambda L - 15\frac{x^2}{L^2}\right)\end{aligned} \tag{19}$$

$$\tau_{zx}^{CR} = -\frac{1}{15}\frac{q_0}{b}\frac{L}{h}\frac{A_0}{C_0}\left(1 - 4\frac{z^2}{h^2}\right)\left(5\frac{x^3}{L^{32}} + \cosh\lambda x - \sinh\lambda x - 1\right) \tag{20}$$

$$\tau_{zx}^{EE} = -\frac{1}{80}\frac{q_0 L}{bh}\left\{\begin{array}{l} \left(1 - 4\dfrac{z^2}{h^2}\right)\left[\begin{array}{l} 40\dfrac{x^3}{L^3} - 8 - \dfrac{2}{3}\dfrac{A_0^2}{C_0}\dfrac{E}{G}\dfrac{h^2}{L^2}\lambda^2 L^2\left(\cosh\lambda x - \sinh\lambda x\right) \\ -\dfrac{B_0}{C_0}\dfrac{E}{G}\dfrac{h^2}{L^2}\left(20\dfrac{x}{L}\right) \end{array}\right] \\ +\dfrac{1}{15}\dfrac{E}{G}\dfrac{h}{L}\dfrac{A_0}{C_0}\left(\dfrac{1}{2}\dfrac{z^2}{h^2} - \dfrac{1}{3}\dfrac{z^4}{h^4} - \dfrac{5}{48}\right)\left(\lambda^2 L^2\left(\cosh\lambda x - \sinh\lambda x\right) - 30\dfrac{x}{L}\right) \end{array}\right\} \tag{21}$$

RESULTS

4.1 Numerical Results

In this research, the results for inplane displacement, transverse displacement, inplane and transverse stresses are presented in the following non dimensional form for the purpose of presenting the results in this work.

For beam subjected to parabolic load, *q(x)*

$$\bar{u} = \frac{Ebu}{q_0 h}, \quad \bar{w} = \frac{10Ebh^3 w}{q_0 L^4}, \quad \bar{\sigma}_x = \frac{b\sigma_x}{q_0}, \quad \bar{\tau}_{zx} = \frac{b\tau_{zx}}{q_0}$$

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Table 1: Non-Dimensional Axial Displacement ($\bar{u}$) at (x = 0.75L, z = h/2), Transverse Deflection ($\bar{w}$) at (x = 0.75L, z =0.0) Axial Stress ($\bar{\sigma}_x$) at (x = 0.0, z = h/2) Maximum Transverse Shears Stresses $\bar{\tau}_{zx}^{CR}$ at (x= 0.01L, z =0.0) and $\bar{\tau}_{zx}^{EE}$ at (x =0.01L, z =0.0) of the Fixed Beam Subjected to Parabolic Load for Aspect Ratio 4.

Source	Model	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
Present	V order	1.6551	0.3219	11.8584	1.1233	-1.4702
Ghugal and Sharma	HPSDT	1.6730	0.3207	12.4566	1.1614	-2.6453
Krishna Murty	HSDT	1.6740	0.3205	11.0920	1.0818	-1.5434
Timoshenko	FSDT	1.5375	0.2781	4.8000	0.0394	2.0403
Bernoulli-Euler	ETB	1.5375	0.1563	4.8000	—	2.0403

Table 4.4: Non-Dimensional Axial Displacement ($\bar{u}$) at (x = 0.75L, z = h/2), Transverse Deflection ($\bar{w}$) at (x = 0.75L, z =0.0) Axial Stress ($\bar{\sigma}_x$) at (x = 0.0, z = h/2) Maximum Transverse Shears Stresses $\bar{\tau}_{zx}^{CR}$ at (x= 0.01L, z =0.0) and $\bar{\tau}_{zx}^{EE}$ at (x =0.01L, z =0.0) of the Fixed Beam Subjected to Parabolic Load for Aspect Ratio 10.

Source	Model	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
Present	V order	24.3175	0.1834	47.9821	4.2549	2.9211
Ghugal and Sharma	HPSDT	24.3646	0.1834	49.5291	4.4978	1.9961
Krishna Murty	HSDT	24.3646	0.1834	46.1200	4.2329	2.0563
Timoshenko	FSDT	24.0234	0.1758	30.0000	0.6165	5.1007
Bernoulli-Euler	ETB	24.0234	0.1563	30.0000	—	5.1007

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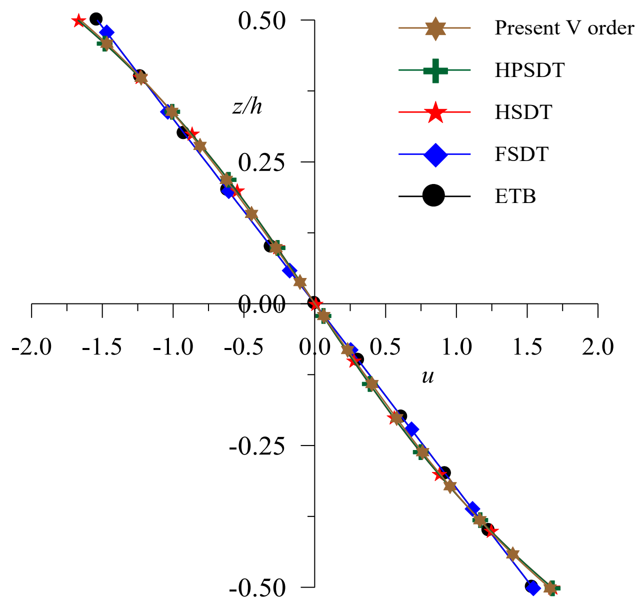

Fig. 3: Variation of axial displacement ($\bar{u}$) through the thickness of fixed-fixed beam at (x = 0.75L, z) when subjected to parabolic load for aspect ratio 4.

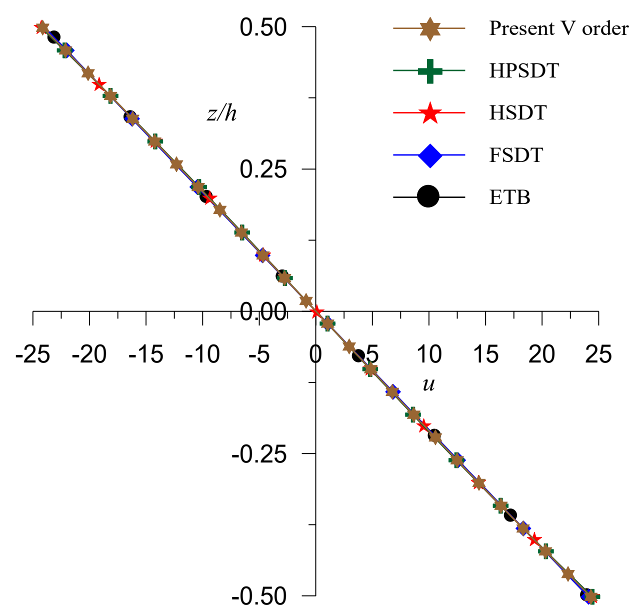

Fig. 4: Variation of axial displacement ($\bar{u}$) through the thickness of fixed-fixed beam at (x = 0.75L, z) when subjected to parabolic load for aspect ratio 10.

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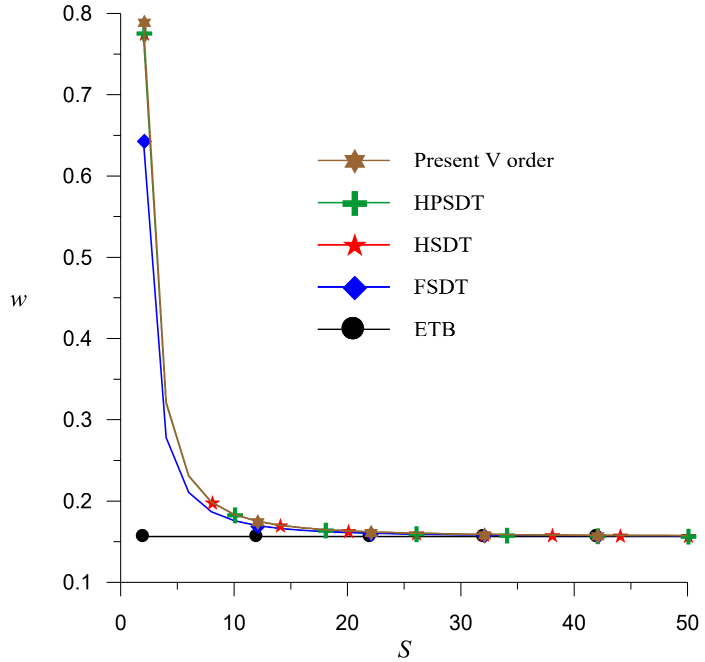

Fig. 5: Variation of maximum transverse displacement ($\bar{w}$) of fixed-fixed beam at (x = 0.75L, z = 0) when subjected to parabolic load with aspect ratio S.

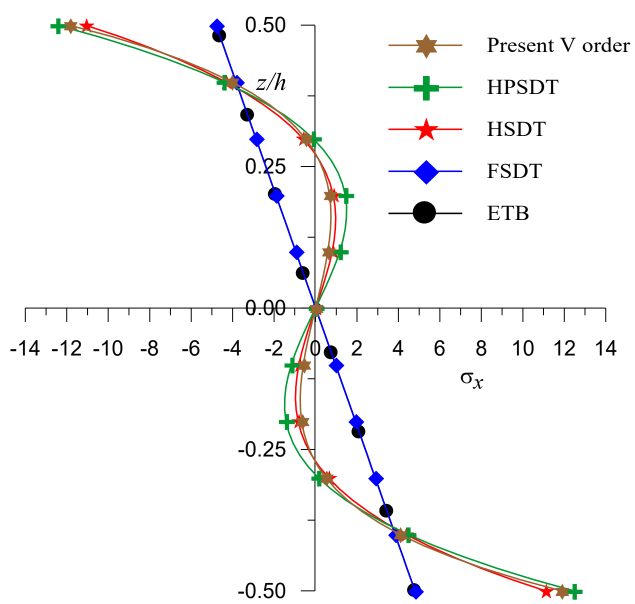

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Fig. 6: Variation of axial stress ($\bar{\sigma}_x$) through the thickness of fixed-fixed beam at (x = 0, z) when subjected to parabolic load for aspect ratio 4.

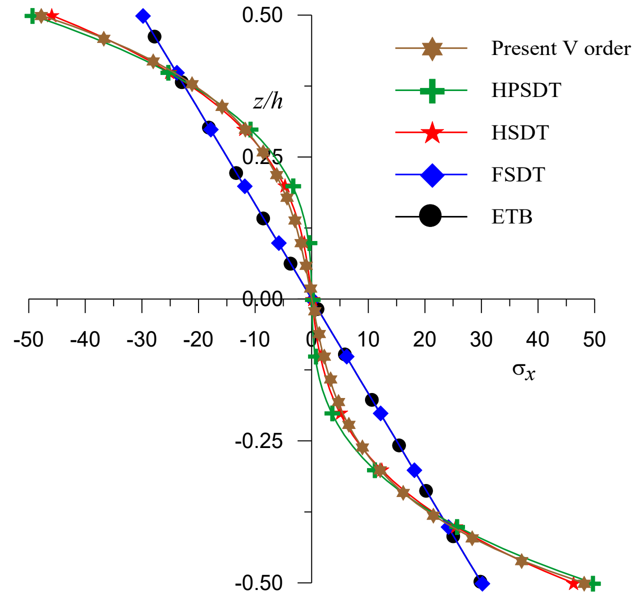

Fig. 7: Variation of axial stress ($\bar{\sigma}_x$) through the thickness of fixed-fixed beam at (x = 0, z) when subjected to parabolic load for aspect ratio 10.

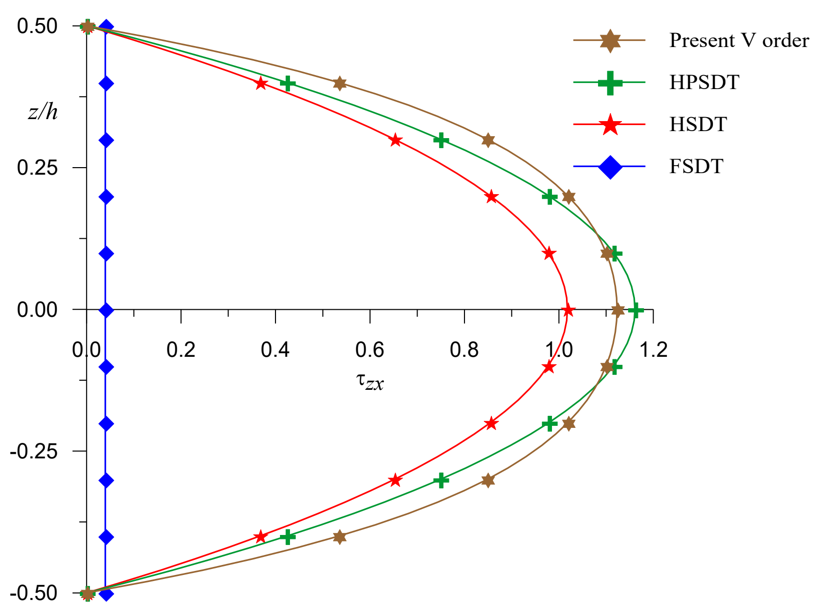

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Fig. 8: Variation of transverse shear stress ($\bar{\tau}_{zx}$) through the thickness of fixed-fixed beam at (x = 0.01L, z) when subjected to parabolic load and obtain using constitutive relation for aspect ratio 4.

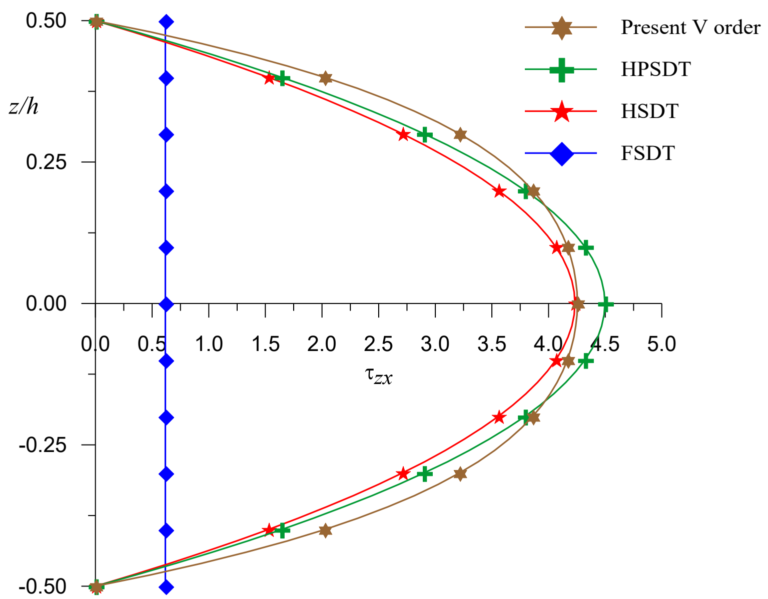

Fig. 9: Variation of transverse shear stress ($\bar{\tau}_{zx}$) through the thickness of fixed-fixed beam at (x = 0.01L, z) when subjected to parabolic load and obtain using constitutive relation for aspect ratio 10.

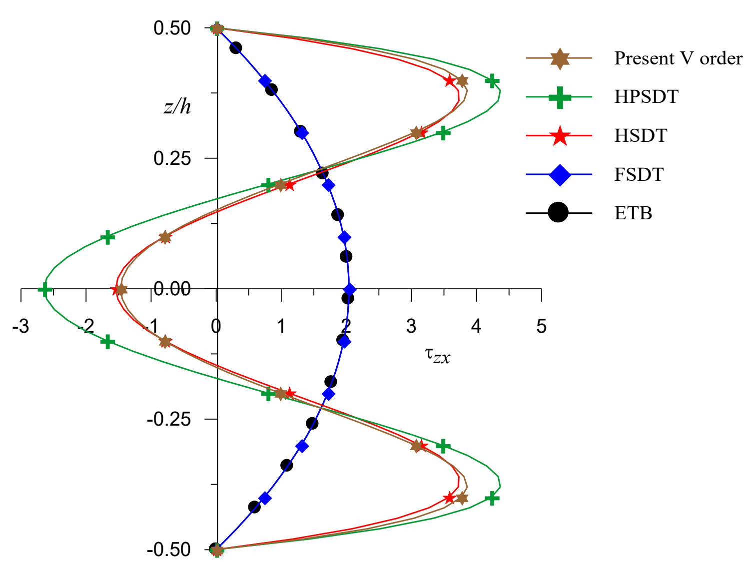

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Fig. 10: Variation of transverse shear stress ($\bar{\tau}_{zx}$) through the thickness of fixed-fixed beam at (x = 0.01L, z) when subjected to parabolic load and obtain using equilibrium equation for aspect ratio 4.

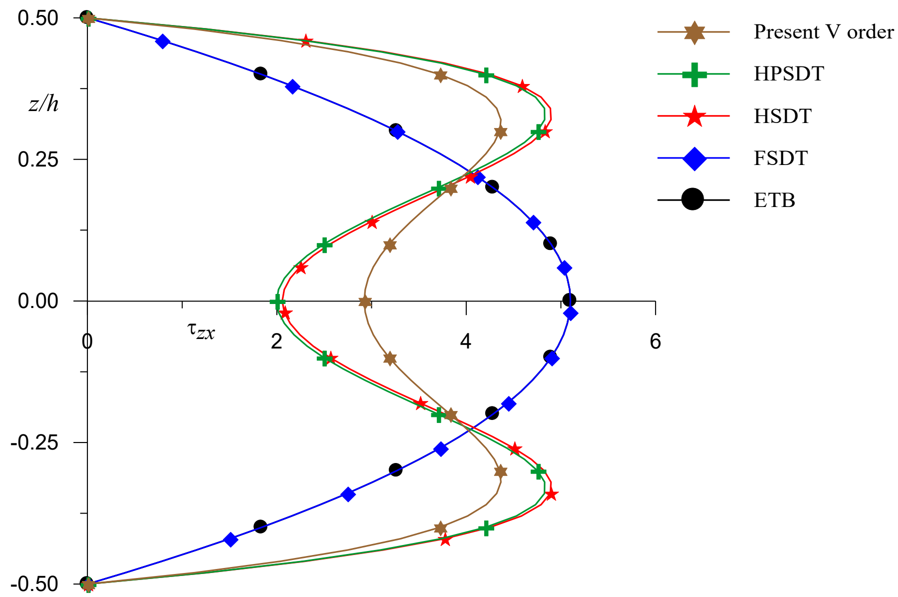

Fig. 11: Variation of transverse shear stress ($\bar{\tau}_{zx}$) through the thickness of fixed-fixed beam at (x = 0.01L, z) when subjected to parabolic load and obtain using equilibrium equation for aspect ratio 10.

4.2 Discussion of Results

The comparison of results of maximum non-dimensional axial displacement ( ) for the aspect ratios of 4 and 10 is presented in Tables 1 and 2 for fixed beams subjected to parabolic load (see Figs. 2). The values of axial displacement given by present theory are in close agreement with the values of other refined theories for aspect ratio 4 and 10. The through thickness distribution of this displacement obtained by present theory is in close agreement with other refined theories except the one given by classical and first order shear deformation theory (FSDT) as shown in Figs. 3 and 4 for aspect ratio 4 and 10. In case fixed beam with parabolic loading, this displacement component shows drastic change in its behavior as can be seen from Fig. 3 at the location x=0.75L for aspect ratio 4, however, this behavior is found to be reversed for aspect ratio 10. The results of

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maximum non-dimensional transverse displacement ( ) for the aspect ratios of 4 and 10 is presented, and it is found that, among the results of all the other theories, the values of present theory are in excellent agreement with the values of other refined theories for aspect ratio 4 and 10 except those of classical beam theory (ETB) and FSDT of Timoshenko. The variation of with aspect ratio (S) is shown in Figure 5. For the aspect ratios greater than 20 all the refined theories converges to the values of classical beam theory.

The results of axial stress ( ) are shown in Tables 1 and 2 for aspect ratios 4 and 10. The axial stresses given by present theory are compared with other higher order shear deformation theories. It is observed that the results by present theory are in excellent agreement with other refined theories. However, ETB and FSDT yield lower values of this stress as compared to the values given by other refined theories. The through the thickness variation of this stress given by ETB and FSDT is linear. Present and other higher order refined theories provide the non-linear variations of axial stress across the thickness at the built-in end due to heavy stress concentration. However, this effect of local stress concentration cannot be captured by lower order theories such as ETB and FSDT. The variations of this stress are shown in Figs. 6 and 7. The maximum transverse shear stress obtained by present theory using constitutive relation is in good agreement with that of TSDT, however HPSDT shows little departure from these theories for aspect ratio 4 and for aspect ratio 10 results of present theory and HSDT are in excellent agreement with each other. Among the values of this stress, the values obtained by HPSDT using equilibrium equation show considerable departure from the values of present and HSDT. The values of present theory and those of HSDT are in good agreement with each other. The through thickness variation of this stress obtained via constitutive relation are presented graphically in Figs. 8 and 9 and those obtained via equilibrium equation are presented in Figs. 10 and 11. It can be seen from these figures that the nature of variation obtained by both the approaches is different from each other.

The through thickness variation of this stress via equilibrium equation shows the anomalous behavior (changes its sign) due to heavy stress concentration associated with the built-in end of the beam. It is seen that the anomalous behavior in the vicinity

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of built-in end cannot be captured by constitutive relation. Further, lower order theories, ETB and FSDT, cannot predict this behavior even with the use of equilibrium equation. Hence, the use of higher order or equivalent shear deformation theories is necessary to recover the effects of stress concentration at the built-in end of the beam with the use of equilibrium equation of two-dimensional theory of elasticity.

CONCLUSIONS

i) The axial displacement ($\bar{u}$)

The present theory gives realistic results of this displacement component in commensurate with the other shear deformation theories. For beams with various loads, the result of present theory is nearly matching with those other higher order theory.

ii) The transverse deflection ($\bar{w}$)

The transverse deflection given by present theory is in excellent agreement with that of other higher order shear deformation theories.

iii) The axial stress ($\bar{\sigma}_x$)

The axial stress and its distribution across the thickness given by present theory is in excellent agreement with that of higher order shear deformation theories.

iv) The transverse shear stresses $\bar{\tau}_{zx}^{CR}$ and $\bar{\tau}_{zx}^{EE}$

Transverse shear stress and its distribution through the thickness of beam obtained from constitutive relation are in close agreement with that of other higher order refined theories; however, use of constitutive relation cannot predict the effect of stress concentration at the built-in end of the beam. The effect of stress concentration on variation of transverse shear stress is exactly predicted by the present theory with the use of equilibrium equation of two dimensional elasticity. Hence the use of equilibrium equation is inevitable to predict the effect of stress concentration in accordance with the higher / equivalent refined shear deformation theories.

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