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Simulation of Flow over V-Notches with Application of Density, Gravity, Head and Notch Angle

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ABSTRACT

This study simulates flows through triangular notches and weirs in a controlled water channel using both experimental measurements and dimensional analysis. The governing parameters considered were the head of flow (H), Density of water (ρ), acceleration due to gravity (g), notch angle (θ), and discharge (Q). By applying the Buckingham's-π theorem, two dimensionless groups were derived, establishing a functional relationship between discharge and the dependent parameters. Experiments were carried out in a laboratory channel to compare observed discharges with those predicted by the mathematical model. The results showed that flows increased non-linearly with head. Variation with angle indicated that discharge peaked at 45°, suggesting the presence of an optimum notch geometry. Model performance was evaluated using statistical indices. The coefficient of determination (r^2=0.9992) indicated near-perfect agreement between observed and predicated values, while the Nash-Sutcliffe efficiency (NSE = 0.88) and Relative BAIS (rBAIS = 0.15) confirmed very good predictive ability with minimal systematic error. The study concludes that the triangular notch is a reliable, cost-effective, and accurate device for measuring low discharges in open channels. Its integration of theoretical modeling, dimensional analysis, and experimental validation provides a strong foundation for both academic research and practical water resources and management applications.

KEYWORDS: *Mathematical modelling, Discharge, Head, Acceleration due to*

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gravity, Angle of Notch

INTRODUCTION

Background of the Study

The design and analysis of systems that manage water resources heavily relies on hydraulic engineering. The precise monitoring of flow, especially in open channels, is one essential component. Effective irrigation and drainage systems, flood control, water resource planning, and environmental monitoring are all supported by flow measurement (Chow, 1959; Subramanya, 2009). Because of their ease of use, affordability, and versatility in a range of field circumstances, notches and weirs have become popular among the many instruments used for measuring flow (Khurmi & Gupta, 2005).

Underestimating discharge during peak flow can have disastrous consequences in flood-prone areas. Inadequate flow control systems are unable to stop flooding downstream or provide early warning. Floods have the power to devastate infrastructure, uproot populations, and destroy crops (Abiola & Jimoh, 2012). Inaccurate flow data from improperly built or calibrated triangular notches might lessen the efficacy of flood prevention measures. Due to its sensitivity at low flow rates, the triangular or V-notches and weirs in particular is quite useful for measuring tiny discharges (Ranga, 2006).

Ideal flow conditions, such as uniform approach velocity, smooth channel walls, and minimal energy losses, are frequently assumed in theoretical models. These don't take into consideration actual disruptions like sediments and turbulence. Despite different notch geometries and heads, an excessive dependence on constant values mainly ignores unstable or sporadic flow situations (Kumar & Yadav, 2015, Mishra & Mohanty, 2010). When theoretical models are used in field settings, these oversimplifications violate the accuracy of discharge forecasts. The head-discharge relationship, which is frequently improved by empirical modeling, can be used to theoretically characterize the flow across such notches. This work provides a straightforward yet effective tool for assessing open channel flows by using the Buckingham's Pi theorem to create a dimensionless mathematical model that links discharge to flow head, gravity, water density, and notch angle (Singh, 1992; Gupta, 2006).

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MATERIALS AND METHODS

Materials

The materials used in course of this work involve the following:

1. Hydraulic bench

2. Five sets of triangular notches.

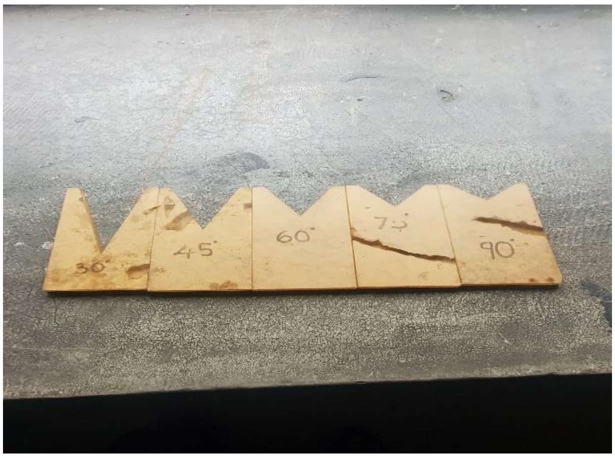

Plate 1: Five sets of Triangular Notches

3. Open flow channel.

Plate 2: Front and Side Views of Flow over notch along the channel

4. Stop watch.

5. Water flow tracer.

6. Protractor.

7. Microsoft office

8. Rectangular channel (made of transparent acrylic or glass for visibility).

9. Sump tank

10. Point gauge or hook gauge (to measure water head above the notch).

11. Hydrometer

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Plate 3: Hydrometer

12. Vernier Caliper.

Plate 4: Vernier Caliper

13. Digital balance or graduated cylinder (for volume measurement accuracy).
14. Camera or mobile phone (to record experimental setups for documentation).

Methods

Experimental Procedures:

- Setup Verification: Ensure the notch is level and the channel base is horizontal.
- Notch Installation: Insert one of the triangular notch plates (starting with 30°).
- Water Supply Initiation: Start the pump and gradually fill the channel until a steady flow is

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established over the notch.

- Head Measurement: Record the head (H) above the notch crest using the point gauge.
- Flow measurement: collect the outflow over a measured time interval (e.g., 30 seconds).
- Determine the volume using a graduated container or balance.
- Data was collected for atleast 4 to 5 times for each notch angle to ensure a reliable comparative dataset.
- Calculate discharge:

$$Q = \frac{Volume\ of\ water\ collected}{Time\ taken} \qquad (1)$$

- Repeat for Multiple Heads: Adjust the inflow to vary the head and repeat the measurements.

Change Notch Angle: Replace with next notch (60°, then 90°) and repeat the entire procedure.

Mathematical Simulation Model

$Q = F_1 (H, g, \rho, \theta)$

Where, Q, H, g, ρ, θ are the discharge, head over the notch, acceleration due to gravity, density and angle of notch respectively.

Determine the dimensions of each variable:

1. Q: L^3T^{-1}
2. H: L
3. g: LT^{-2}
4. ρ: ML^{-3}
5. θ: $M^0L^0T^0$ (dimensionless)

Applying the **Buckingham Pi Theorem:**

The Buckingham Pi theorem states that the number of π terms is equal to the number of variables minus the number of fundamental dimensions. In this case, we have 5 variables and 3 fundamental dimensions (M, L, T), but θ is dimensionless, so we expect 2 π-terms (5 - 3 = 2).

Form the π terms by combining variables:

We have non-repeating variables: Q and θ.

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We have repeating variables:

For geometric properties: H

For flow properties: g

For fluid properties: ρ

Now π_1 and $\pi_2 = M^0 L^0 T^0$ (2)

$\pi_1 = H^a g^b \rho^c . Q = M^0 L^0 T^0$

Replacing variables with their dimensions

$(L)^a (LT^{-2})^b (ML^{-3})^c . L^3 T^{-1} = M^0 L^0 T^0$

$L^a L^b T^{-2b} M^c L^{-3c} L^3 T^{-1} = M^0 L^0 T^0$

Collect like terms:

$M^c L^{a+b} T^{-2b-1} = M^0 L^0 T^0$

M: c = 0

T: -2b - 1 = 0

$\frac{-2b}{-2} = \frac{1}{-2}$

b = $-\frac{1}{2}$

L: a + b -3c + 3 = 0

a + $(-\frac{1}{2})$ – 3(0) + 3 = 0

a - $\frac{1}{2}$ + 3 = 0

a = - $\frac{5}{2}$

$\pi_1 = H^{-5/2} g^{-1/2} \rho^0 . Q$

$= H^{-5/2} g^{-1/2} . Q$

$\pi_1 = \frac{Q}{H^{5/2} g^{1/2}}$ (3)

So, $\pi_2 = H^a g^b \rho^c . \theta = M^0 L^0 T^0$

$= (L)^a (LT^{-2})^b (ML^{-3})^c . M^0 L^0 T^0 = M^0 L^0 T^0$

$= L^a L^b T^{-2b} M^c L^{-3c} = M^0 L^0 T^0$

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Take like terms;

$M^{c}T^{-2b}L^{a+b-3c}$

M: c = 0

T: $\frac{-2b}{-2} = \frac{0}{-2}$

b = 0

L: a + b – 3c

a + 0 – 3(0) = 0

a = 0

$\pi_2 = H^0 g^0 \rho^0 . \theta$

$\pi_2 = \theta$ (4)

Express the relationship between the π terms:

$F_2(\pi_1, \pi_2) = 0$

This can be rewritten as:

$\pi_1 = \lambda . \varphi (\pi_2)$

Our model will become:

$\frac{Q}{H^{5/2} g^{1/2}} = \lambda(\tan(\frac{\theta}{2}))^k$ (5)

This relationship can be used to model discharge (Q) in terms of the other variables.

Performance Matrix or Goodness of fit test

In engineering hydraulics, a performance matrix or goodness-of-fit test is used to evaluate the accuracy and reliability of a model or simulation. It compares predicted values with observed or measured data.

Performance Metrics to be used;

1. rBIAS: Relative Bias

The Relative Bias is a statistical performance metric used to evaluate how well a model or measurement system estimates a known reference value. It measures the systematic error; that is, the average tendency of a model to overestimate the true value across multiple observations.

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Formula:

$$rBIAS = \frac{1}{N}\sum\left(\frac{|Q_{obv}(i) - Q_{sim}(i)|}{Q_{obv}(i)}\right) \qquad (6)$$

Where;

Q_{OBV} = Observed flow in (m^3/s).

Q_{SIM} = Simulated flow in (m^3/s).

N = Number of data points.

2. Nash-Sutcliffe Efficiency (NSE)

The Nash-Sutcliffe Efficiency coefficient assesses how well a model is predicted values match the observed (measured) values. It compares the predictive power of your model to the performance of simply using the mean of the observed values.

Formula:

$$NSE = 1 - \frac{\sum_{i=1}^{n}(Q_{SIM}(i) - Q_{OBV}(i))^2}{\sum_{i=1}^{n}(Q_{OBV}(i) - \bar{Q})^2} \qquad (7)$$

Where;

Q_{SIM} = Predicted or simulated flow in (m^3/s) at a time or point *i*.

Q_{OBV} = Observed flow (measured) in (m^3/s) value at *i*

$\bar{Q}$ = Mean of observed values.

n = Number of data points.

3. Coefficient of Determination

The Coefficient of Determination (R^2) is a statistical measure that indicates how well the predicted values from a model correspond to the observed data. It represents the proportion of variance in the observed values that is explained by the model. An R^2 value of 1 implies a perfect fit, while a value near 0 suggests poor predictive ability. In hydraulic modeling, a high R^2 demonstrates that the model accurately captures the relationship between head and discharge. The coefficient of determination (r^2) is calculated using the ratio:

$$R = \frac{n\sum_{i=1}^{n}\left(Q_{i,obs} \cdot Q_{i,sim}\right) - \sum_{i=1}^{n}\left(Q_{i,obs}\right)\sum_{i=1}^{n}\left(Q_{i,sim}\right)}{\sqrt{n\sum_{i=1}^{n}\left(Q_{i,obs}{}^2\right) - \left[\sum_{i=1}^{n}\left(Q_{i,obs}\right)\right]^2} \cdot \sqrt{n\sum_{i=1}^{n}\left(Q_{i,sim}{}^2\right) - \left[\sum_{i=1}^{n}\left(Q_{i,sim}\right)\right]^2}} \qquad (8)$$

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RESULTS AND DISCUSSION

Table 1 represents the geometric parameters

(Head of flow in meters), flow parameters (Acceleration due to gravity in meters) and fluid parameters (Density of water = 1000kg/m^3) and the discharge (in m^3/s) of each notch (in degrees) throughout the experiment.

Table 1: Geometric, Fluid and Flow Parameters

Experiments	**Discharge, Q (m^3/s)**	**Head of Flow, H(m)**	**Acceleration Due to Gravity g (m/s^2)**	**Notch angle (degrees)**	**Density ρ (kg/m^3)**
1	0.000401754	0.0585	9.81	30	1000
2	0.000432857	0.051	9.81	45	1000
3	0.000151123	0.029	9.81	60	1000
4	0.000144927	0.026	9.81	75	1000
5	0.000130617	0.023	9.81	90	1000

The table of geometric, fluid, and flow parameters highlights the fundamental variables governing discharge through the triangular notch. The geometric property, represented by the head of flow (H), was the primary variable influencing discharge. As the head increased, the effective potential energy available for flow also increased, leading to higher discharge values. The fluid property, water density (ρ), remained constant under laboratory conditions but was essential in scaling the discharge equation, particularly in dimensional analysis. The flow property, acceleration due to gravity (g), provided the driving force that converted potential energy at the head to kinetic energy at the notch.

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Head of flow against Notch Angle

Figure 1 shows the direct relationship between the head of flow and the angle of the notch.

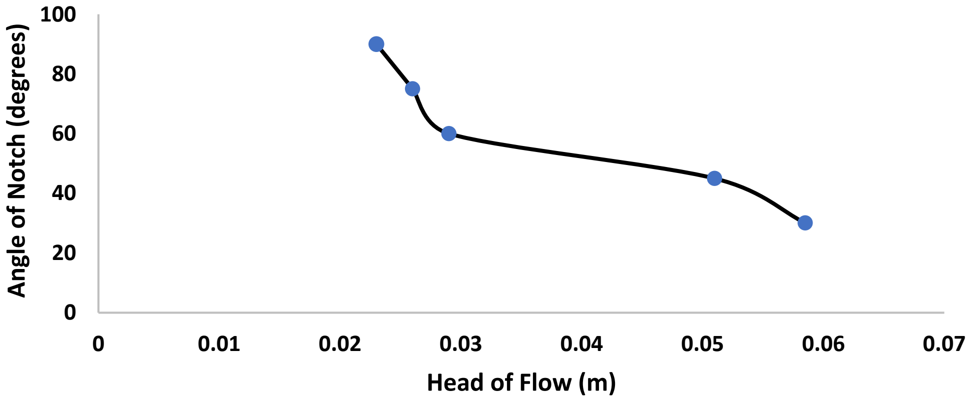

Figure 1: Head of flow against Angle of Notch

From the graph it is observed that the head of flow increases as the angle of the notch reduces. It is also seen that there was a drastic increase in the head of flow from the 60° notch to the 45° notch. The relationship between head of flow (H) and notch angle (θ) revealed an inverse trend, as the notch angle decreased, the head required to achieve a given discharge increased. Narrower notch openings (e.g., 30°) restricted the flow area, which meant that a higher head was necessary to drive water through the notch compared to wider angles (e.g., 90°). Consequently, more head is needed to compensate for the smaller flow area and achieve the same discharge. In practical terms, this indicates that sharper triangular notches are more sensitive for low flows, but the require larger heads to pass higher discharges. This makes them particularly useful for accurately measuring small flows in irrigation and laboratory applications.

Discharge against Notch Angle

Figure 2 shows the relationship between the discharge and the angle of the notches. From the graph, the discharge gradually increased from the 90° notch to the 60° notch and then peaked at the 45° notch.

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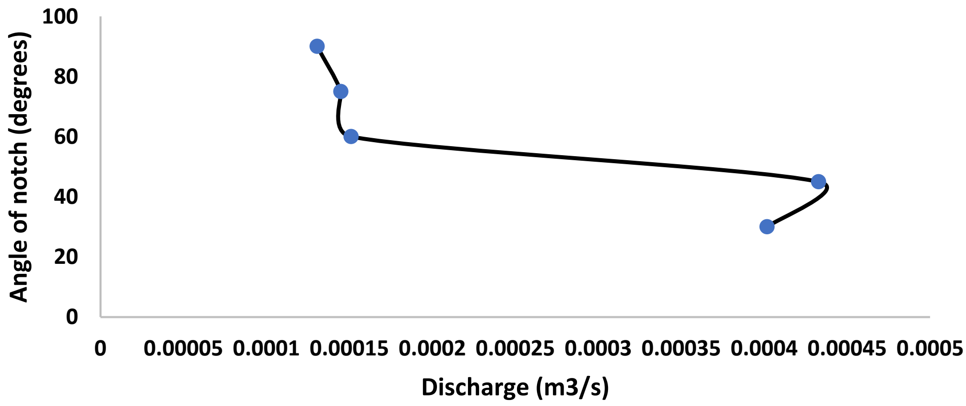

Figure 2: Discharge against Angle

The variation of discharge with notch angle showed that discharge increased as the angle was reduced, with the maximum value observed around the 45° notch. At larger angles, such as 90°, the notch allowed a broader flow area, but the reduced head sensitivity caused lower discharges compared to narrower angles. Conversely, at very small angles (below 45°), the notch restricted the flow area, and despite the higher head, the discharge began to decline. This trend suggests that there exists an optimum notch angle where the competing effects of head sensitivity and flow area balance to produce the highest discharge. In this experiment, the optimum was observed at approximately 45°. This finding agrees with studies in hydraulics that note sharper notches improve sensitivity at low flows, while moderate angles can maximize discharge efficiency.

Head of flow against Discharge

Figure 3 demonstrates the direct relationship between the head of flow and the discharge, showing how the discharge increases as the flow head increases, and decreases as the flow head decreases.

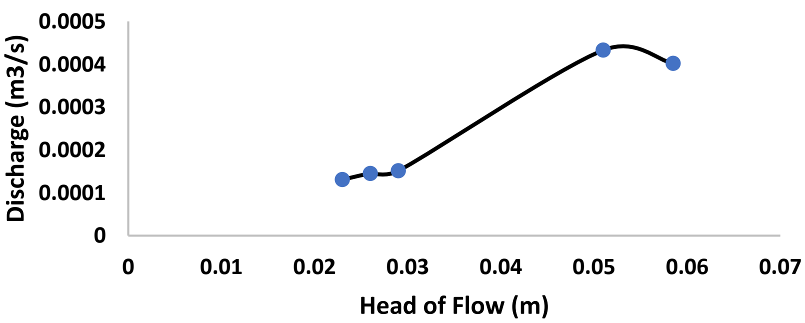

Figure 3: Head of flow against Discharge

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The relationship between head of flow (H) and discharge (Q) showed a direct and nonlinear trend, with discharge rising sharply as the head increased. At lower heads, increases in discharge were modest, but as the head grew larger, the rate of increase in discharge became more pronounced due to the power dependence. This confirms that the head of flow is the dominant geometric parameter controlling discharge through the notch. Practically, it means that small errors in measuring head can lead to significant variations in predicted discharge, highlighting the need for precision in head measurement.

Simulated Results of Discharge

Simulated discharge was compared with the observed values obtained from the experiment, and the results are seen in the figure below.

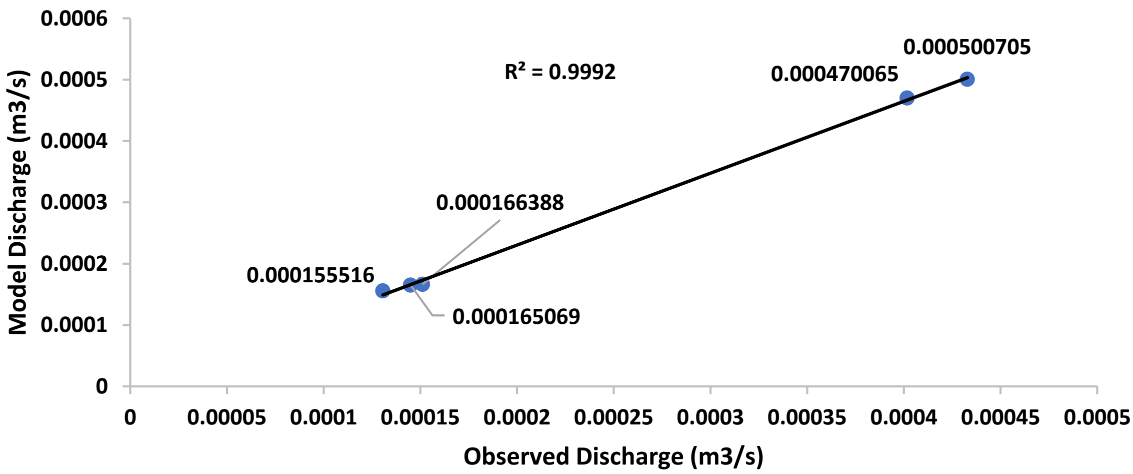

Figure 4: A graph of Observed Discharge against Simulated Discharge

To further examine the relationship between observed and simulated discharges, the results were plotted on a linearized graph. Recall from figure 4.3, that the observed discharge is directly proportional to the head of flow. This means that the data points were expected to fall along a straight line.

The plotted graph confirmed this expectation, both observed and simulated discharges showed strong linearity with high correlation, and the two points nearly overlapped. Minor deviations of the observed values from the theoretical line were attributed to experimental errors such as turbulence at the notch and measurement inaccuracies.

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To calculate the simulated discharge, we recall eqn. (9):

$$\frac{Q}{H^{5/2}g^{1/2}} = \lambda.(\tan(\frac{\theta}{2}))^k \qquad (9)$$

We take the natural log of both sides

$$ln\frac{Q}{H^{5/2}g^{1/2}} = ln\{\lambda.(tan(\frac{\theta}{2}))^k\}$$

$$= ln\,\lambda + ln(tan(\frac{\theta}{2}))^k$$

$$= ln\,\lambda + k\,ln(tan(\frac{\theta}{2}))$$

$$ln\frac{Q}{H^{5/2}g^{1/2}} = k\,ln(tan(\frac{\theta}{2})) + ln\,\lambda$$

With;

$y = ln\frac{Q}{H^{5/2}g^{1/2}}$

$m = k$

$x = ln(tan(\frac{\theta}{2}))$

$c = ln\,\lambda$

Where;

K = m = Slope of graph

c = intercept

Our model equation will be;

$$\boldsymbol{Q = 0.6198g^{1/2}H^{5/2}[tan\,(\frac{\theta}{2})]^{0.9821}} \quad \textbf{(10)}$$

Using the model equation, it was possible to compute the values of the simulated discharge for all the five experiments. These values were then plotted alongside the observed discharge, against the experiments as seen in figure 5 below.

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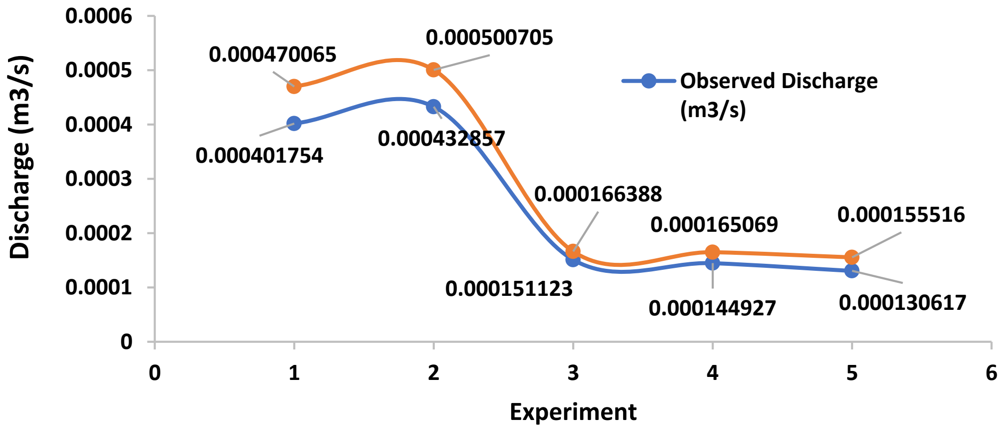

Figure 5: A graph of Observed and Model Discharges

The graph comparing experimental discharge with model discharge shows a very close alignment between the two datasets. Both curves follow the same decreasing trend, and the points for observed discharge lie close to the model line. This indicates that the model equation provides a reliable prediction of actual discharge values. The small difference observed are mainly due to experimental fa0ctors such as turbulence and measurement accuracy, but they remain within acceptable limits. Overall, the graph validates the accuracy of the model in simulating flow through the triangular notch.

Validation of Model

The graph comparing observed and simulated discharge in figure 5 clearly demonstrates a strong agreement, with both following nearly identical trends. This visual consistency is further supported by the goodness-of-fit tests applied.

Table 2 shows the calculation of the goodness-of-fit tests.

Table 2: Goodness-of-fit Test Presentation

			rBIAS	NSE	
Experiment	**Qobs (m3/s)**	**Qsim (m3/s)**	**abs(Qobs-Qsim)/Qobs**	**(Qobs-Qsim)^2**	**(Qobs-Qmean_obs)^2**
1	0.000401754	0.000470065	0.170031388	4.66637E-09	2.23498E-08
2	0.000432857	0.000500705	0.156744709	4.60336E-09	3.26169E-08

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3	0.000151123	0.000166388	0.101013166	2.33032E-10	1.02279E-08
4	0.000144927	0.000165069	0.138980131	4.05699E-10	1.15194E-08
5	0.000130617	0.000155516	0.190625939	6.1996E-10	1.47959E-08
SUM=	0.001261278		0.757395332	1.05284E-08	9.15099E-08
MEAN=	0.000252256		0.151479066		
rBIAS=			**0.151479066**		
NSE=				**0.884947834**	
r^2	**0.9992**				

The model achieved a Nash-Sutcliffe Efficiency (NSE) value of 0.88, which indicates very good agreement between observed and simulated discharges. This shows that the model was able to capture most of the variability in the measured data. The Relative Bias (rBIAS) was 0.15, suggesting that the model predictions were almost unbiased, with only a slight tendency toward overestimation. The coefficient of determination (R^2) value of 0.9992 indicates an almost perfect fit between the observed and predicted discharge values. This shows that the triangular notch model explains nearly all the variability in the data and is highly reliable for predicting flow. Together, these results confirm that the model equation provided reliable simulation of discharge under the experimental conditions.

CONCLUSION AND RECOMMENDATIONS

CONCLUSION

This study successfully simulated flow through triangular notches in a controlled water channel, combining theoretical analysis, dimensional modeling, and experimental validation. The triangular notch proved effective for measuring low discharges, with experimental results closely aligning with model predictions. This study can be concluded as follows:

1. In this study, the geometric property considered was the head of flow (H), the fluid property was the density of water (ρ), and the flow property was the acceleration due to gravity (g). the head of flow was found to directly influence discharge, with increasing head resulting in higher

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flow rates through the triangular notch. The density of water, although relatively constant under laboratory conditions, was essential in establishing the scaling relationships in the discharge equation. The acceleration due to gravity governed the conversion of potential energy into kinetic energy, thereby controlling the velocity component of flow through the notch. Together, these properties determined the functional discharge relationship. This highlights the critical role of geometric, fluid, and flow parameters in accurately simulating and predicting discharge through a triangular notch.

2. The mathematical model developed for the prediction of flow through the triangular notch was based on the principles of dimensional analysis and the established discharge equation. By applying the Buckingham π theorem to the governing variables, head of flow (H), density of water (ρ), acceleration due to gravity (g), notch angle (θ), and discharge (Q). This model confirmed that discharge is strongly dependent on head and notch angle, while being influenced by fluid and flow properties.

3. The performance of the triangular notch model was evaluated using two statistical indices: The Nash-Sutcliffe Efficiency (NSE) and the Relative Bias (rBIAS). The NSE value of 0.88 indicated that the simulated discharges closely matched the observed values, capturing most of the variability in the data. The rBIAS value of 0.15 showed that the model was nearly unbiased, with only a very slight tendency to overestimate flow. a high r^2 value of 0.9992 demonstrates that the model accurately captures the relationship between head and discharge. Together, these metrics confirmed the accuracy of the triangular notch model for predicting discharge under controlled flow conditions.

RECOMMENDATIONS

Test additional notch angles and geometries. Create empirical curves for each angle and geometry and compile a practical lookup table, as it helps practitioners choose the best notch geometry for a target flow range.

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