State Feedback based Stability Augmentation System for Airplane Aviation

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Abstract

This article reports state feedback based stability augmentation system for airplane aviation .The aircraft motion in both longitudinal as well as lateral directions have been considered to improve the dynamic stability of the aircraft. The State feedback gain matrix has been derived using the Bass-Gura algorithm. Aircraft motion in longitudinal and in lateral direction have been studied with and without state feedback gain the designed algorithm is implemented on MATLAB/ SIMULINK software.

Keywords:- *Controllability, Observability, Transformation matrix*

INTRODUCTION

The estimation problem plays an important role in control processes for given input and output for estimation of variables [1], [2], [3], and it has been studied by few authors [4], [5]. In case of difficulty in measuring the state variables, the number of sensor component must be reduced.[1], [2], [3].

In observer related application, the implementation of a closed-loop control algorithm is most important [6]. The idea of the observer for the dynamical process has been proposed by Luenberger [7],[8]. The observer, commonly known as 'Luenberger observer,' came into existence many years after the Kalman filter. Kalman filter is a special type of observer both for the system with input or with unknown inputs used to reduce the order of observers. Time delays associated with observer design have also been studied [9], [10], [11] by the Lyapunov theory deals the observers designed for solving metrical inequalities.

The pole placement controller and observer are both connected to each other for observer problems. For the gain matrix, it is needed to determine a way that poles remain at the desired location. The duality principle for the transformation of controller pole placement to observer pole placement techniques has been applied. With the help of algorithms, the gain matrix can be found, and conversion into their observer can be achieved by controller pole algorithms. [12]. This work deals with the study of a state feedback based stability augmentation system. An attempt has been made to design a state feedback controller using the Bass-Gura technique for pole placement. A general aviation airplane [13], [14] is considered as an example, and complex simulation is performed in MATLAB/ SIMULINK software.

BASS-GURA ALGORITHM FOR STATE FEEDBACK GAIN

A system is completely controllable when a system state can be transferred from an initial state to any other desired state. A system is completely observable when output can be used to identify each state. If one or more states cannot be found in the output state, then the system is not observable. The general expression for the realization of controllability and observability of the system are given below.

$$\dot{X} = A\dot{x} + B\eta \quad (1)$$

$$y = Cx + D\eta \quad (2)$$

where x and η indicate the state and control vectors of order n and m. y indicates the output vectors and A, B, C, D is the constant matrices. There are necessary conditions to satisfy the observability matrices (U) and controllability matrices (V). The general expression of the observability matrix is

$$U = [C^T, A^T C^T, \dots\dots\dots\dots (A^T)^{n-1} C^T] \quad (3)$$

and the general expression for finding Controllability matrix is

V= $[B, AB, A^2B, \ldots\ldots\ldots\ldots, A^{n-1}B]$ (4)

The main aim is to find the gain matrix (k) by Bass Gura techniques, where the feedback gain can be described as

$$k = [(VW)^T]^{-1}[\bar{a} - a] \quad (5)$$

Here V, the controllability matrix and W, the transformation matrix, are the vectors obtained from the coefficient characteristics equation of closed-loop and open-loop systems ($\bar{a}$ and a). In the case of longitudinal stability augmentation, the desired characteristic equations of an unknown feedback system can be found with the help of short-period and long-period roots.

$$\lambda_{1,2} = -\zeta_{sp}\omega_{n_{sp}} \pm i\omega_{n_{sp}}\sqrt{1-(\zeta_{sp})^2} \quad (6)$$

$$\lambda_{3,4} = -\zeta_p\omega_{n_p} \pm i\omega_{n_p}\sqrt{1-(\zeta_p)^2} \quad (7)$$

where ζ_{sp} *and* ζ_p is the damping ratio of short-period and long period whereas $\omega_{n_{sp}}$ *and* ω_{n_p} are the undamped natural frequencies for short-period and long-period. For lateral stability augmentation, the desired characteristic roots depend on spiral approximation, Roll approximation and also Dutch roll approximation.

General expression of characteristic root of spiral equation is

$$\lambda_{spiral} = \frac{L_\beta N_r - L_r N_\beta}{L_\beta} \quad (8)$$

where L_β is a stability derivative due to dihedral effect, N_r is a yaw rate damping, L_r is a roll moment due to yaw rate and N_β is directional stability. Some condition for the stability of spiral is as follows

$$L_\beta N_r - L_r N_\beta > 0 \quad (9)$$

Another characteristics root of lateral motion formed from Roll approximation can be represented as

$$\lambda_{roll} = -1/\tau = L_p \quad (10)$$

In equation (10), L_p magnitude depends upon the size of the wing and tail surfaces.

Characteristic root formed due to Dutch roll approximation is

$$\lambda^2 - \left(\frac{Y_\beta + u_0 N_r}{u_0}\right)\lambda + \frac{Y_\beta N_r - N_\beta Y_r + u_0 N_\beta}{u_0} = 0 \quad (11)$$

STABILITY AUGMENTATION OF GENERAL AIRPLANE AVIATION

The validation for general characteristics of airplane aviation can be performed with the help of the Bass-Gura algorithm, which finds the gain matrix. Here, the stability augmentation of longitudinal and lateral motion of a general airplane is presented, and the validation of the algorithm is performed in MATLAB/Simulink environment.

Aircraft longitudinal motion

The state equation of general airplane aviation [13], [14] for the longitudinal motion is derived and expressed in vector-matrix form as

$$\begin{bmatrix}\Delta\dot{u}\\ \Delta\dot{w}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix}-0.045 & 0.036 & 0 & -32.2\\ -0.369 & -2.02 & 176 & 0\\ 0.0019 & -0.0396 & -2.948 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\end{bmatrix} + \begin{bmatrix}0\\ 28.146428\\ -11.8743\\ 0\end{bmatrix}[\Delta\delta_e]$$

u is the aircraft longitudinal velocity, w is the aircraft attack angle, θ is the aircraft pitch angle, q is the aircraft pitch angular rate, δ_e is the elevator deflection. Δ is associated with the variables from normal values. The output equation is chosen as y = Cx with C= [0010]. The Matlab/Simulink design model is presented in figure 1. The controller gain matrix has been calculated using Bass-Gura algorithms. The characteristic equation of the open-loop system is obtained by solving the equation

$$|\lambda I - A| = 0 \quad (12)$$

After solving the equation we get,

$$\lambda^4 + 5.013\lambda^3 + 13.161404\lambda^2 + 0.669908032 + 0.59410288 = 0 \quad (13)$$

The vector a is created from the coefficients of theopen loop characteristics equation:

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \quad (14)$$

$$a = \begin{bmatrix}a_1\\ a_2\\ a_3\\ a_4\end{bmatrix} = \begin{bmatrix}5.013\\ 13.161404\\ 0.669908032\\ 0.59410288\end{bmatrix}\begin{bmatrix}5.013\\ 13.161\\ 0.670\\ 0.600\end{bmatrix} \quad (15)$$

The next step in the design process is to find the transformation matrix as given below.

$$W = \begin{bmatrix}1 & a_1 & a_2 & a_3\\ 0 & 1 & a_1 & a_2\\ 0 & 0 & 1 & a_1\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (16)$$

From equations (15) and (16), the transformation matrix is

$$W = \begin{bmatrix} 1 & 5.013 & 13.161 & 0.670 \\ 0 & 1 & 5.013 & 13.161 \\ 0 & 0 & 1 & 5.013 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

where W is the transformation matrix.

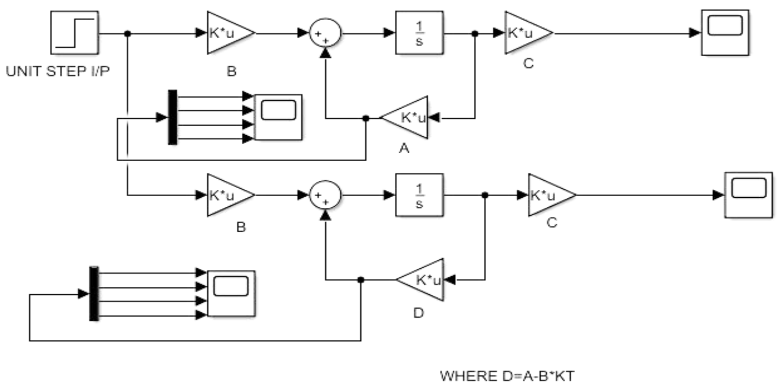

Fig. 1: Block diagram of Simulation used in Matlab software.

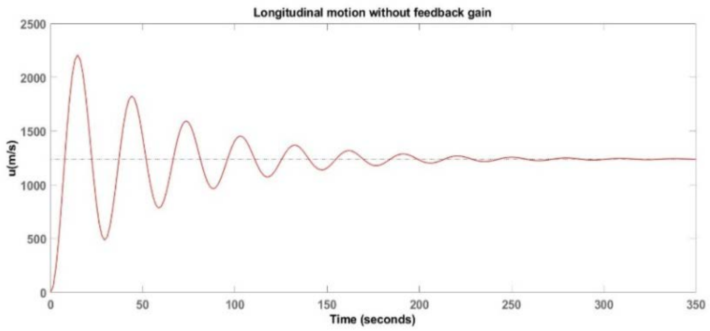

Fig. 2: u(m/s) variation for longitudinal motion without feedback

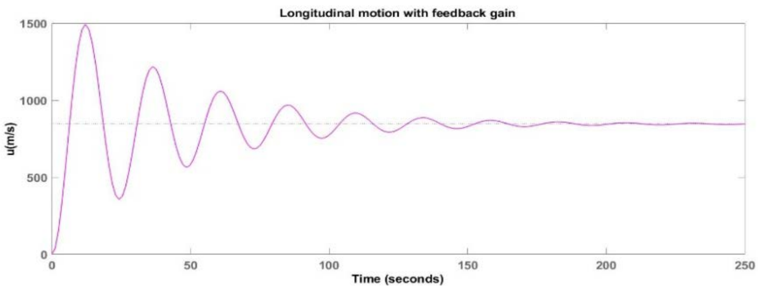

Fig. 3: u(m/s) varitation for longitudinal motion with feedback.

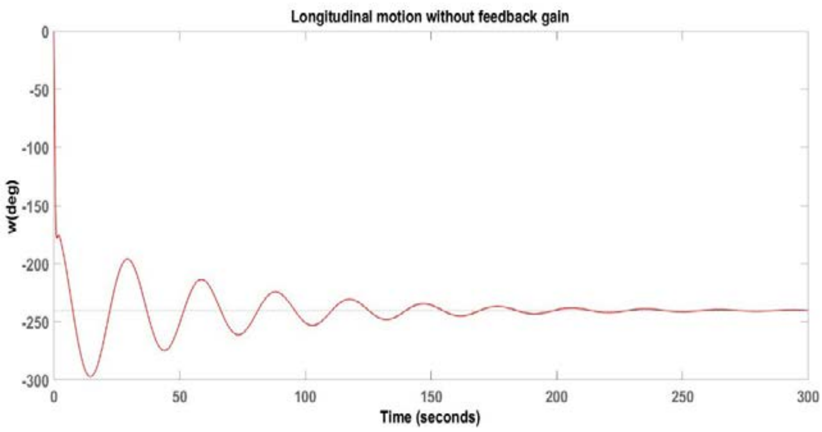

Fig. 4: w(deg) varitation for longitudinal motion without feedback

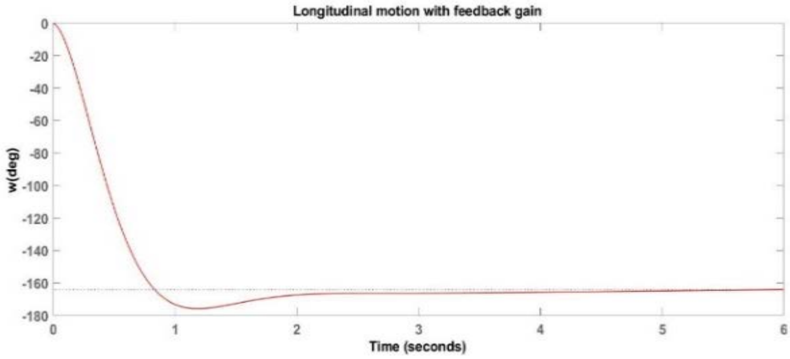

Fig. 5: w (deg) varitation for longitudinal motion with feedback

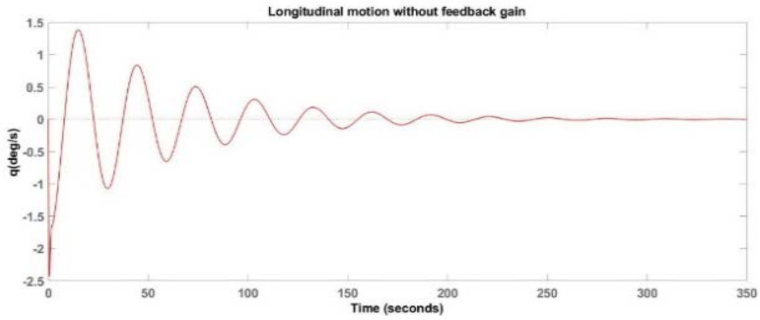

Fig 6: q (deg/s) variation for longitudinal motion without feedback

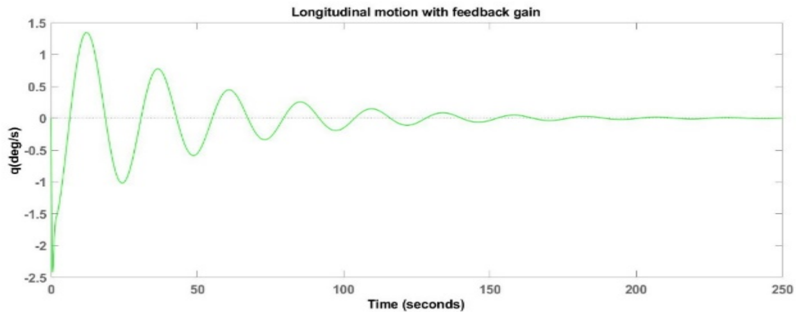

Fig. 7: q(deg/s)variation for longitudinal motion with feedback

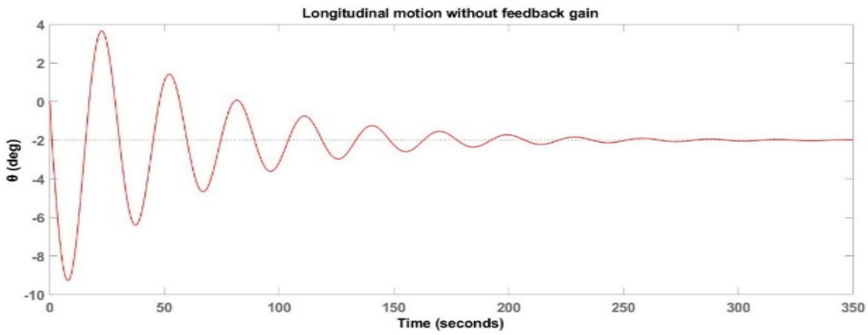

Fig. 8: θ(deg)varitation for longitudinal motion without feedback

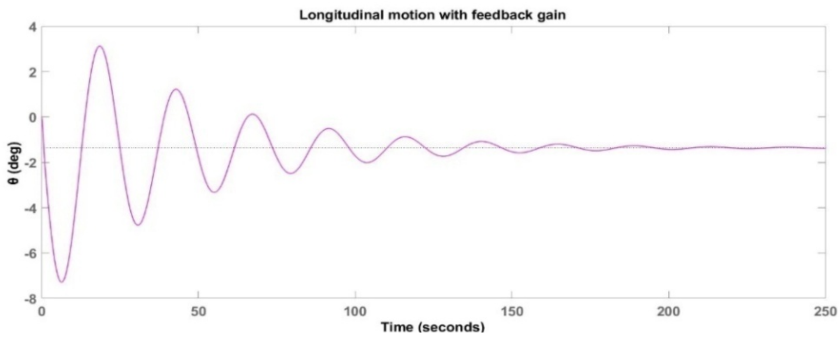

Fig. 9: θ(deg)varitation for longitudinal motion with feedback

Aircraft lateral motion

For the lateral motion of the aircraft [13],[14],the obtained state equation is represented in matrix form as

$$\begin{bmatrix}\Delta\dot{v}\\ \Delta\dot{p}\\ \Delta\dot{r}\\ \Delta\dot{\emptyset}\end{bmatrix}=\begin{bmatrix}-0.254 & 0 & -1.0 & 0.182\\ -16.02 & -8.40 & 2.19 & 0\\ 4.488 & -0.350 & -0.760 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta v\\ \Delta p\\ \Delta r\\ \Delta\emptyset\end{bmatrix}+\begin{bmatrix}0 & 12.4478571\\ -28.916305 & 23.08988\\ -0.224229 & -4.6127\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\delta_a\\ \Delta\delta_r\end{bmatrix}$$

where v is the aircraft sideslip angle, r is the aircraft yaw angular rate, p is the aircraft roll angular rate, $\emptyset$ is the aircraft roll angle, δ_r is the rudder deflection and δ_a is the ailerons deflection. The eigen values which are related to airplane stability augumented system must be chosen for motion in lateral directions. Eigen values can be determined from the characteristics equation

$$|\lambda I - A| = 0 \qquad (18)$$

After solving the equation,

$$\lambda^4 + 9.417\lambda^3 + 13.982\lambda^2 + 48.102\lambda + 0.4205 = 0 \qquad (19)$$

$$\lambda_{spiral} = -0.144s^{-1} \qquad (20)$$

$$\lambda_{roll} = L_p = -8.4s^{-1} \qquad (21)$$

$$\lambda_{DR} = -0.51 \pm 2.109i \qquad (22)$$

After simplification, the equation can be written in the form of expression $\lambda^2 + 1.102\lambda + 4.71 = 0$ (23)

The desired characteristic equation using eigenvalues obtained above can be expressed as

$$(\lambda + 0.144)(\lambda + 8.4)(\lambda^2 + 1.102\lambda + 4.71) = 0 \qquad (24)$$

After solving the equation, the following equation is obtained

$$\lambda^4 + 9.646\lambda^3 + 15.335088\lambda^2 + 41.5752192\lambda + 5.697216 = 0 \qquad (25)$$

Comparing equation (19) with equation (26) , the equation obtained is as follows,

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \qquad (26)$$

$$\text{Considering } a=\begin{bmatrix}a_1\\ a_2\\ a_3\\ a_4\end{bmatrix}=\begin{bmatrix}9.417\\ 13.982\\ 48.102\\ 0.4205\end{bmatrix} \qquad (27)$$

Comparing equation (25) with equation (28), the following equation is obtained,

$$\lambda^4 + \overline{a_1}\lambda^3 + \overline{a_2}\lambda^2 + \overline{a_3}\lambda + \overline{a_4} = 0 \qquad (28)$$

$$\text{Assuming } \bar{a}=\begin{bmatrix}\overline{a_1}\\ \overline{a_2}\\ \overline{a_3}\\ \overline{a_4}\end{bmatrix}=\begin{bmatrix}9.646\\ 15.335088\\ 41.5752192\\ 5.697216\end{bmatrix} \qquad (29)$$

$$W=\begin{bmatrix}1 & 9.417 & 13.982 & 48.102\\ 0 & 1 & 9.417 & 13.982\\ 0 & 0 & 1 & 9.417\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad (30)$$

where w is the transformation matrix.TheSimulink design model for bass gura algorithm is shown in figure 10.

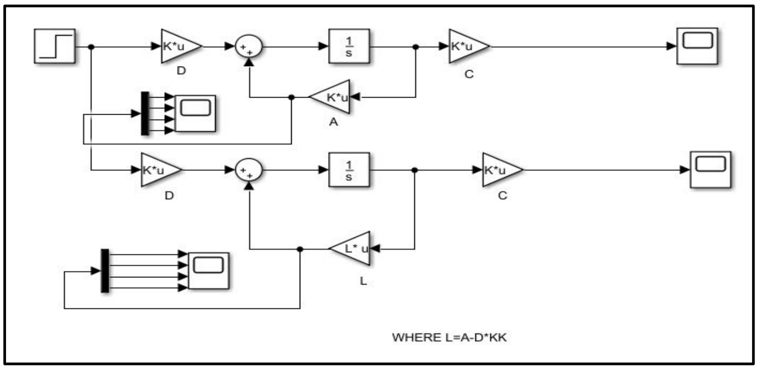

Fig. 10: Block diagram of simulation in matlab software.

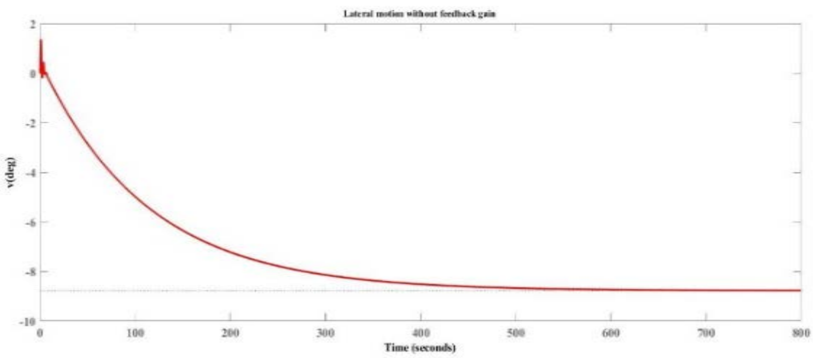

Fig. 11: v(deg) variation for lateral motion without feedback

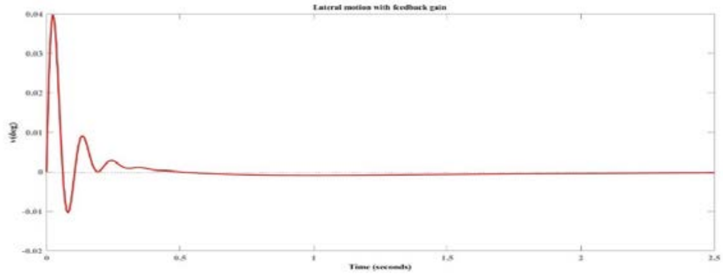

Fig. 12: v(deg) variation for lateral motion with feedback

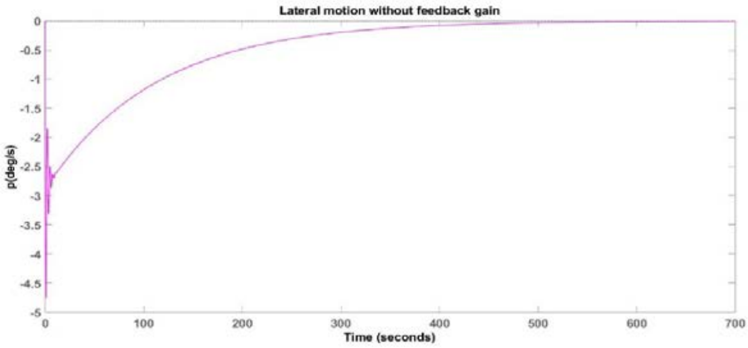

Fig. 13:P(deg/s) variation for lateral motion without feedback

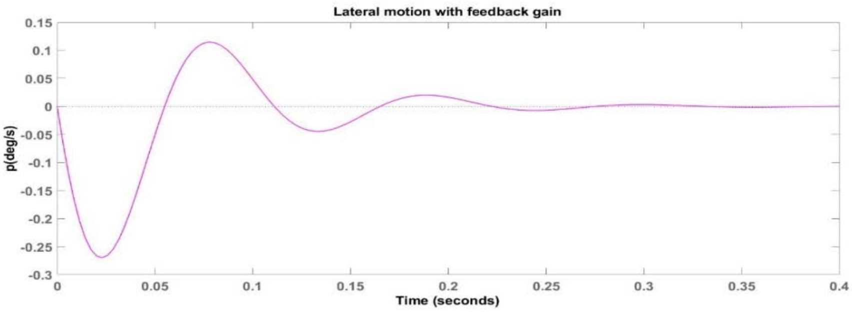

Fig. 14: p(deg/s) variation for lateral motion with feedback

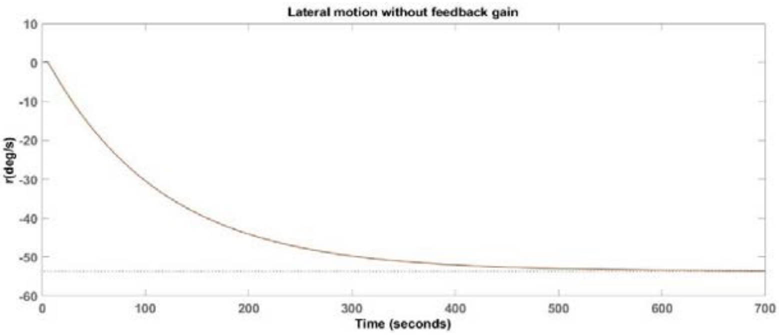

Fig. 15: r(deg/s) variation for lateral motion without feedback

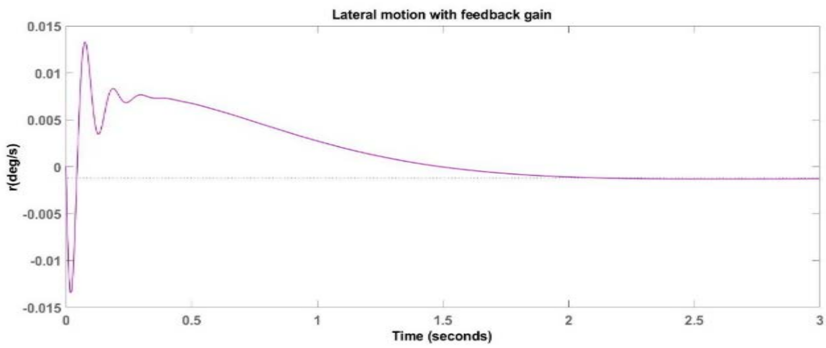

Fig. 16: r(deg/s) variation for lateral motion with feedback

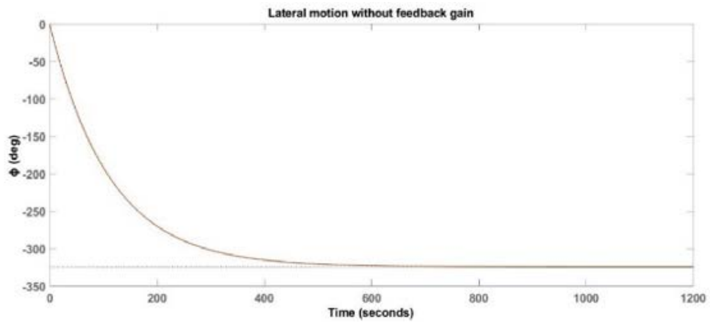

Fig. 17: ∅ (deg) variation for lateral motion without feedback

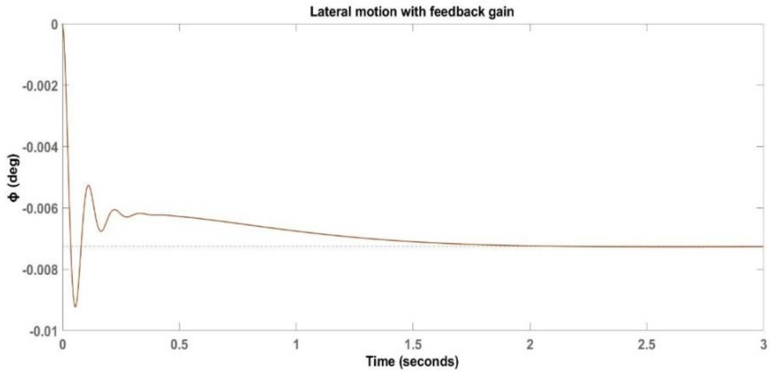

Fig. 18: ∅ (deg) variation for lateral motion with feedback

RESULT AND DISCUSSION

Table1shows longitudanal motion parameters while Table 2 provides lateral motion results.

Table 1: Longitudinal Motion Results

PARAMETERS	WITHOUT FEEDBACK GAIN	WITH FEEDBACK GAIN	Remarrks
u(m/s)	Fig.(2	Fig.(3)	Improvement of dynamic stability
w(deg)	Fig.(4)	Fig.(5)	
q(deg/s)	Fig.(6)	Fig.(7)	
θ(deg)	Fig.(8)	Fig.(9)	

Table 2: Lateral Motion Results

PARAMETERS	WITHOUT FEEDBACK GAIN	WITH FEEDBACK GAIN	Remarrks
v(deg)	Fig.(11)	Fig.(12)	Improvement of dynamic stability
P(deg/s)	Fig.(13)	Fig.(14)	
r(deg/s)	Fig.(15)	Fig.(16)	
ϕ(deg)	Fig.(17)	Fig.(18)	

v indicates the aircraft sideslip angle, p indicates the aircraft roll angular rate, r indicates the aircraft yaw angular rate, and ϕ indicates the aircraft roll angle. It is observed that the dynamic stability of the aircraft in longitudinal (from fig. 2 to fig. 9) and in lateral directions (from fig. 11 to fig. 18) has been improved with the inclusion of state feedback.

CONCLUSION

State feedback based stability augmentation system for airplane aviation system is presented in this work. State feedback controller is designed for longitudinal and lateral motion of aircraft. State feedback gain matrix is derived using the Bass-Gura algorithm. From the result, it is observed that the dynamic stability of the aircraft both in lateral and longitudinal motion with state feedback gain is improved.

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